# Perspectives on Aerodynamic Shape Optimization

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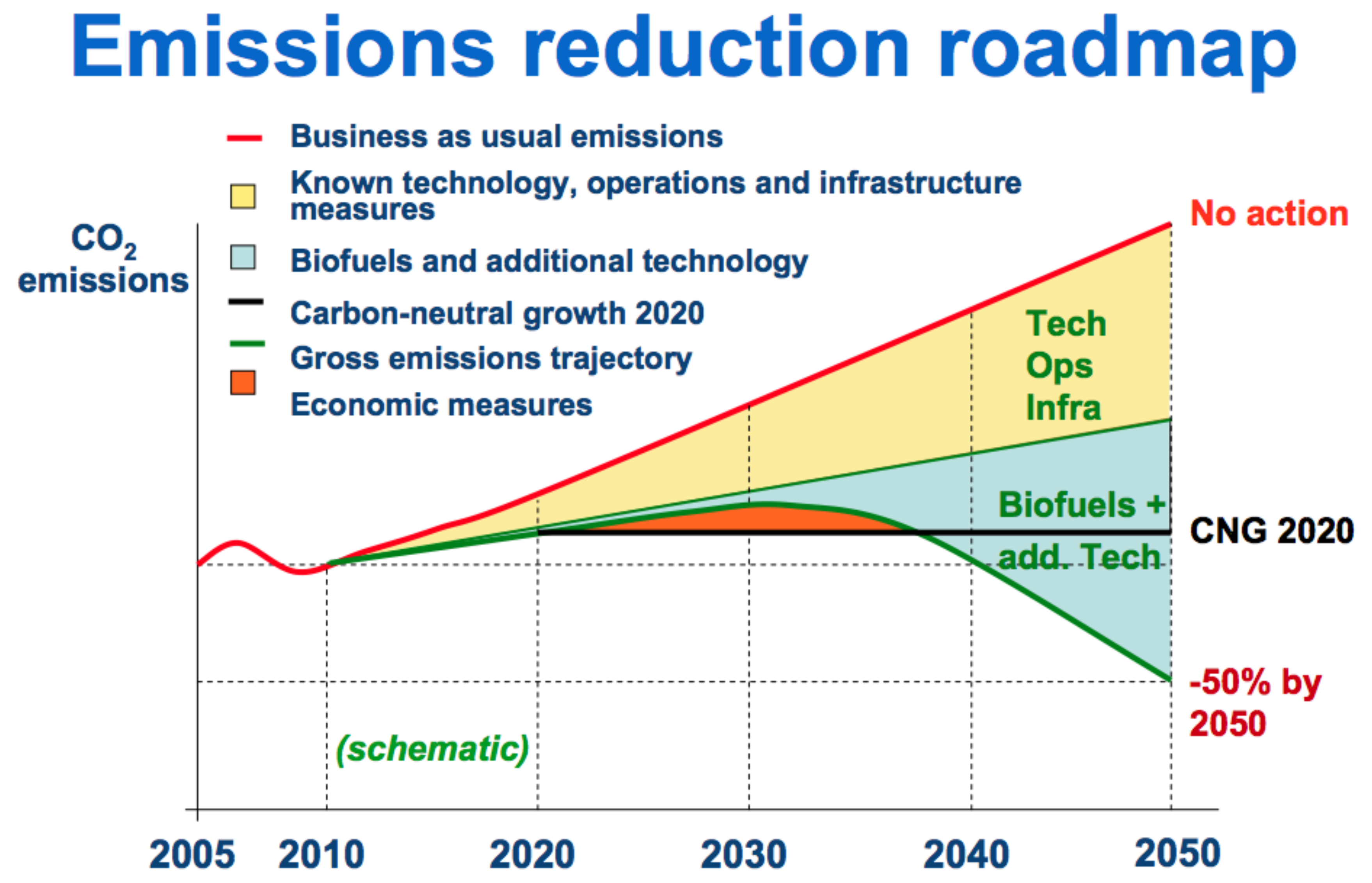
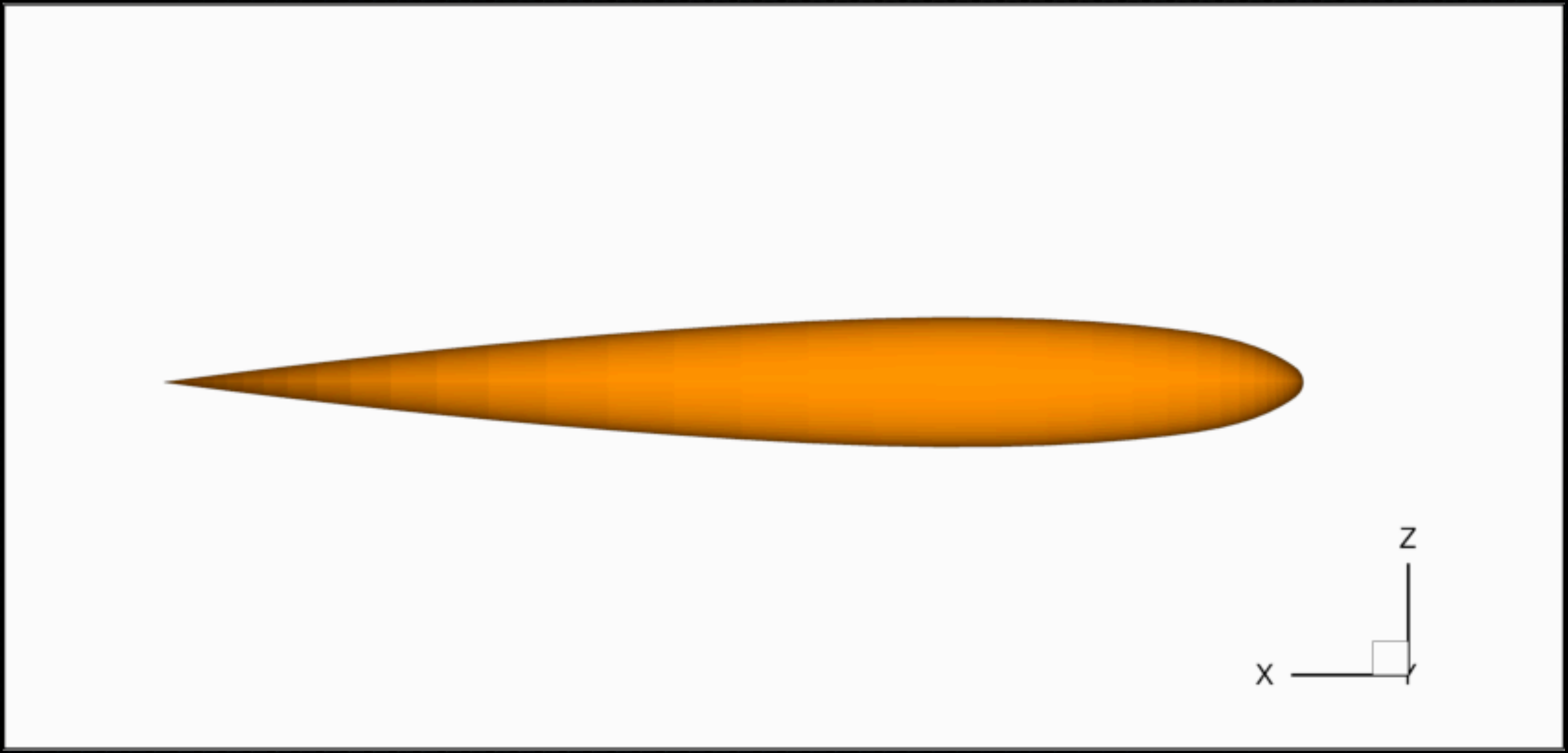
# MOTIVATION



Emissions reduction roadmap

# Application to Wing Design

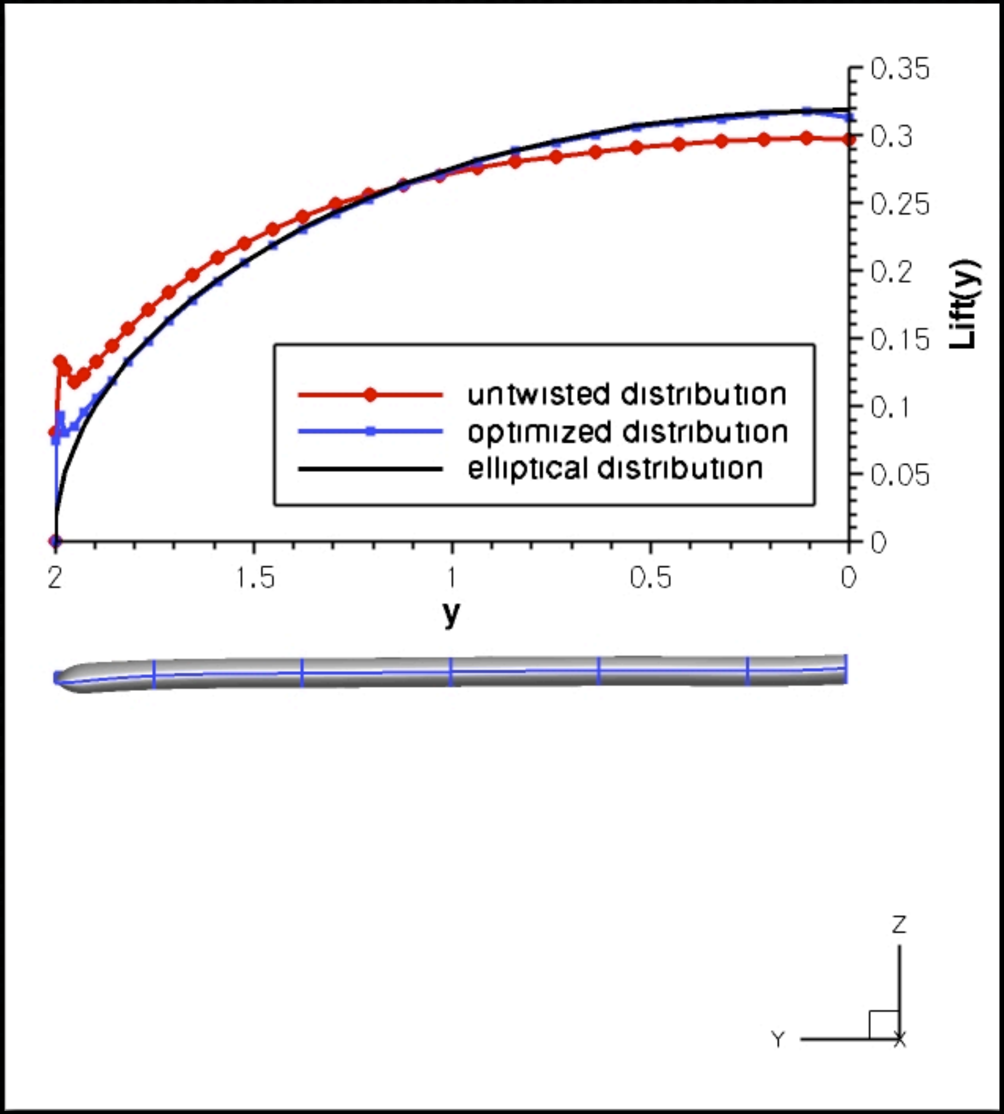
## Lift-constrained induced-drag minimization



# Twist Optimization

**Validation**: recover elliptical lift distribution using twist

- 6 sections free to move
- rectangular wing
- constrained lift

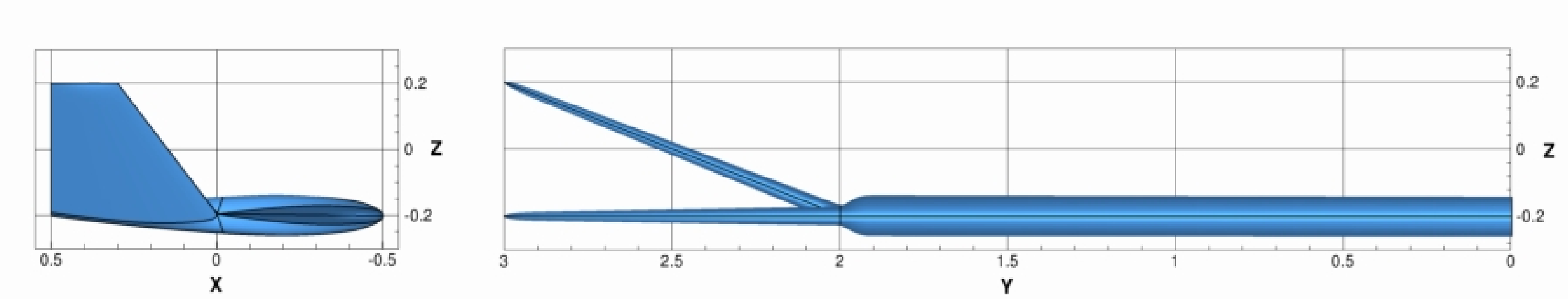


# High-Fidelity Aerodynamic Shape Optimization

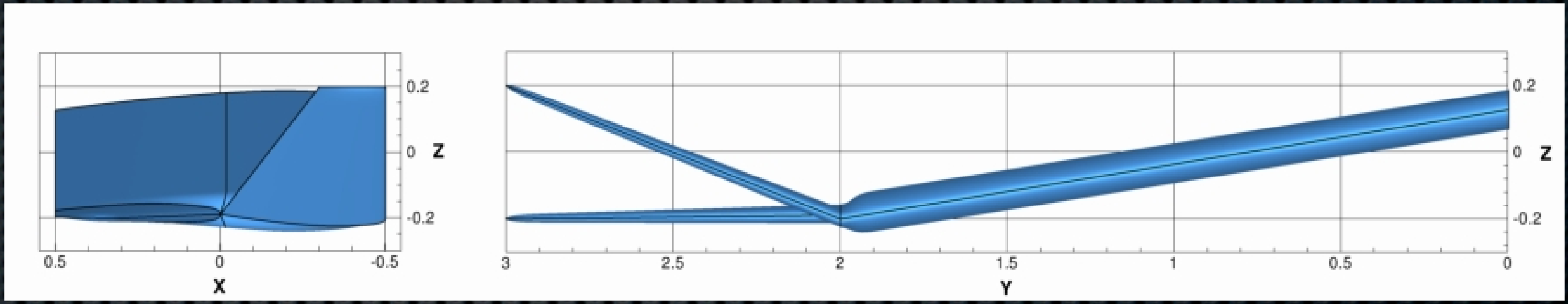
- a component within high-fidelity multi-disciplinary optimization (MDO)
- high-fidelity physics, e.g. Euler or Reynolds-averaged Navier-Stokes equations
- incremental optimization is preceded by conceptual and preliminary design using lower fidelity tools
- exploratory optimization permits large shape changes and could be used to uncover new concepts

# The Split Tip Wing

- down-up configuration: span efficiency = 1.159



- up-down configuration: span efficiency = 1.167



- Hicken, J.E., and Zingg, D.W., Induced Drag Minimization of Nonplanar Geometries Based on the Euler Equations, AIAA Journal, Vol. 48, No. 11, 2010

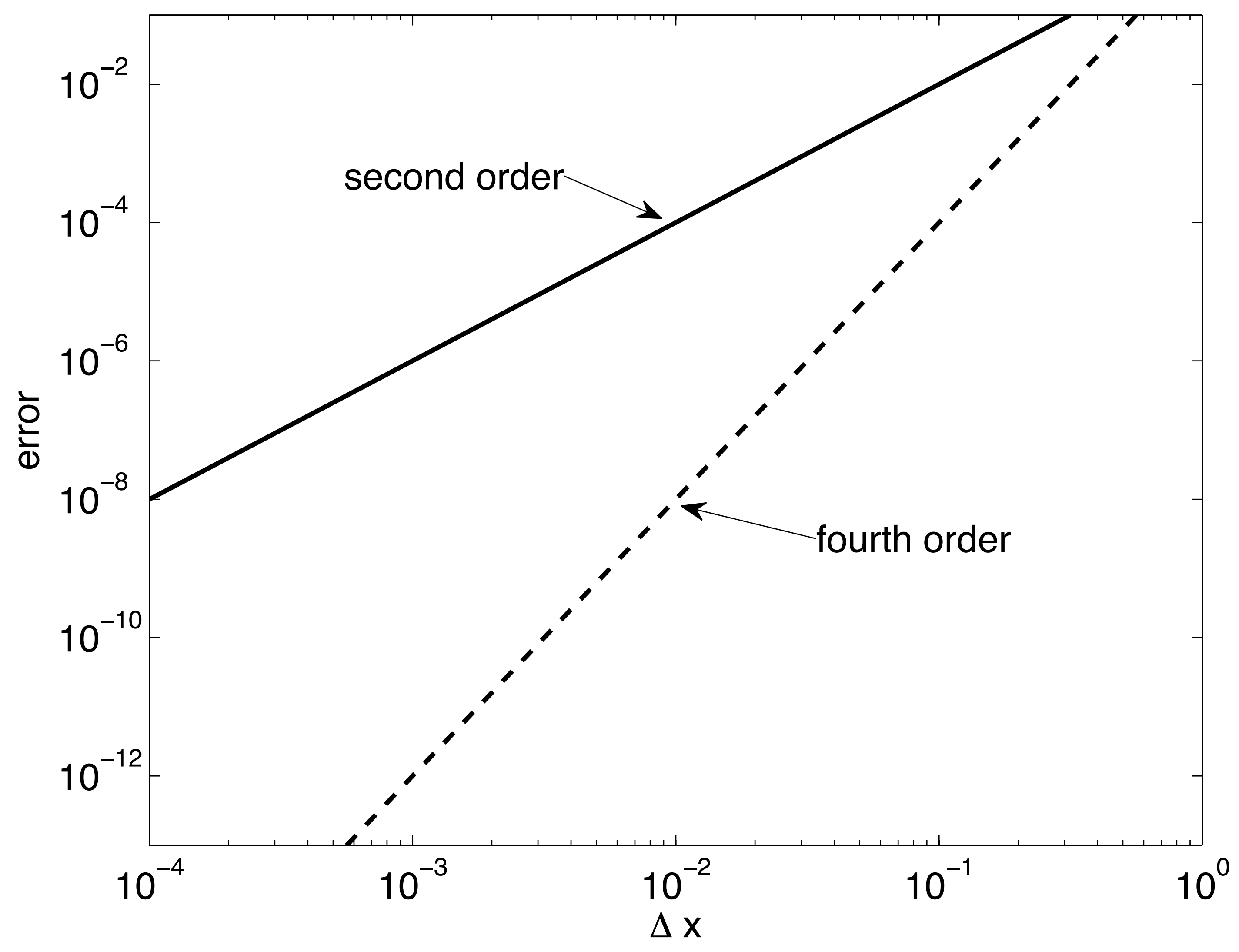
# Topics

- Computational fluid dynamics
  - ★ higher order?
  - ★ finite-difference methods? structured grids?
  - ★ summation by parts, dual consistency, and superconvergence
  - ★ parallel Newton-Krylov-Schur algorithm
- Geometry parameterization, mesh movement, adjoint method
- Problem formulation: range of operating conditions, multiple constraints
- Choice of optimization algorithm: multimodality in aerodynamic shape optimization

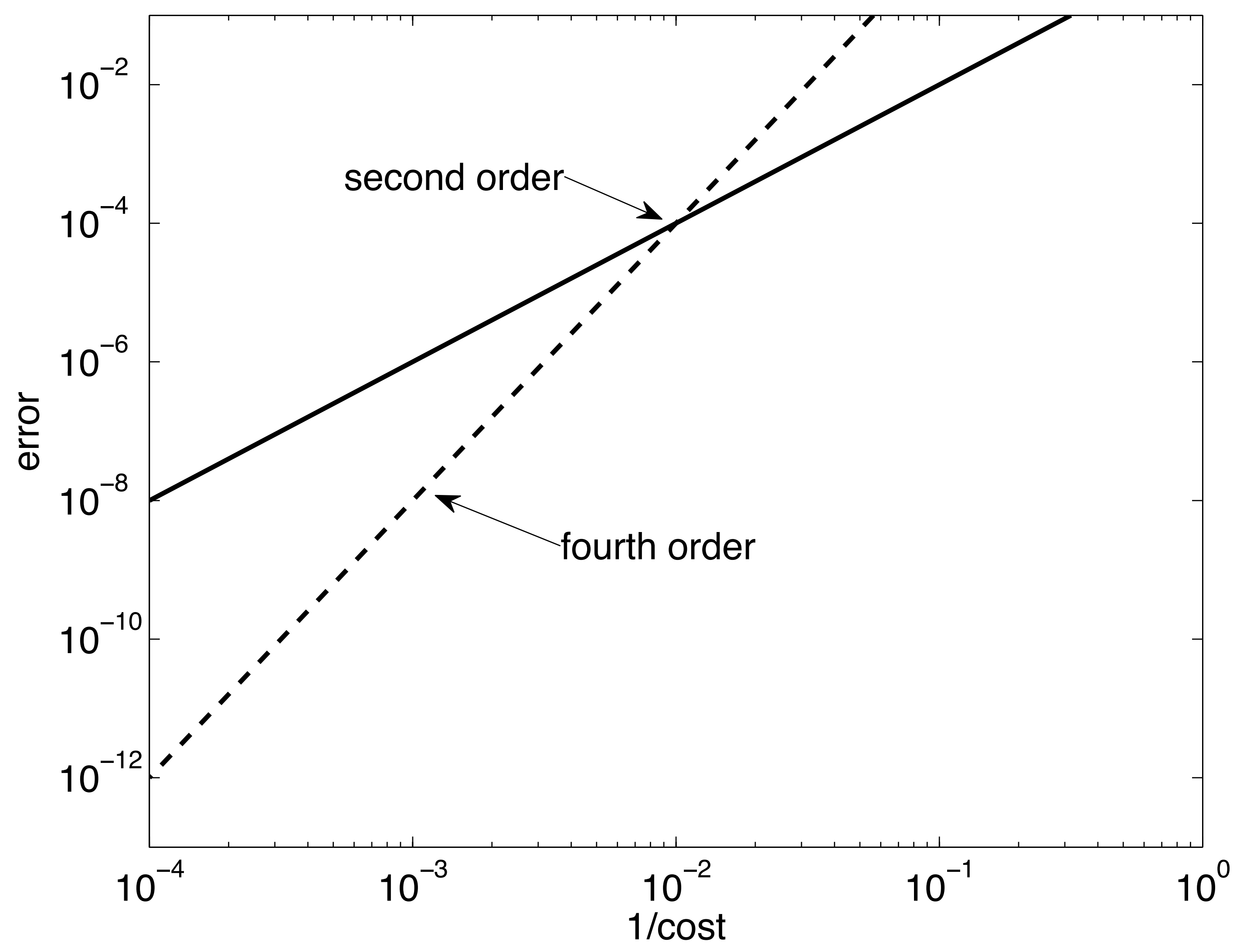
# CFD: 2nd or higher order?

- conventional wisdom: higher order is advantageous for applications like LES, DNS, CAA, which are very demanding in terms of mesh resolution
- actually can be advantageous in any context where low error is required
- higher order shown to be more efficient than second order for steady RANS computations by De Rango and Zingg, AIAA J., Vol. 39, 2001
- DPWs show that computing drag on a 3D configuration is very demanding in terms of mesh resolution

# Error vs mesh spacing



# Error vs computational cost



# Higher order methods

- higher order generally not achieved in practical problems due to shocks, singularities, discontinuities, etc.
- numerical error can nevertheless be lower on a given mesh
- current interest is concentrated on discontinuous Galerkin methods

# Structured or unstructured meshes?

- conventional wisdom: unstructured meshes are easier to generate and superior for adaptation; hence pursue higher-order DG schemes
- however, higher-order finite difference methods on structured meshes are much more efficient than higher-order methods for unstructured meshes
- is the former advantage sufficient to outweigh the latter disadvantage?

# Summation-by-Parts (SBP) Operators

- Satisfy a discrete summation-by parts property that mimics the continuous operator
- Used in combination with simultaneous approximation terms (SATs) at boundaries
- Rigorous development of time-stable boundary schemes for higher-order methods
- Superconvergent functional estimates if scheme is dual consistent
  - ➡ For example, the fourth-order scheme produces sixth-order convergence in functionals

➡ Hicken, J.E., and Zingg, D.W., Superconvergent Functional Estimates from Summation-by-Parts Finite-Difference Discretizations, SIAM Journal on Scientific Computing, Vol. 33, 2011

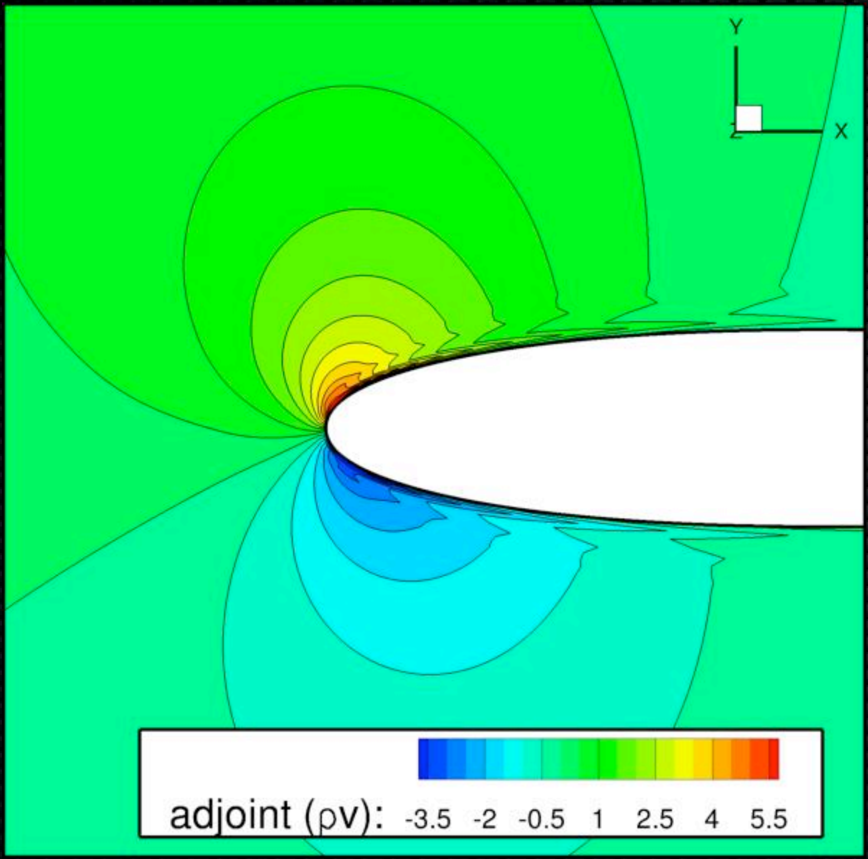
# Dual Consistency

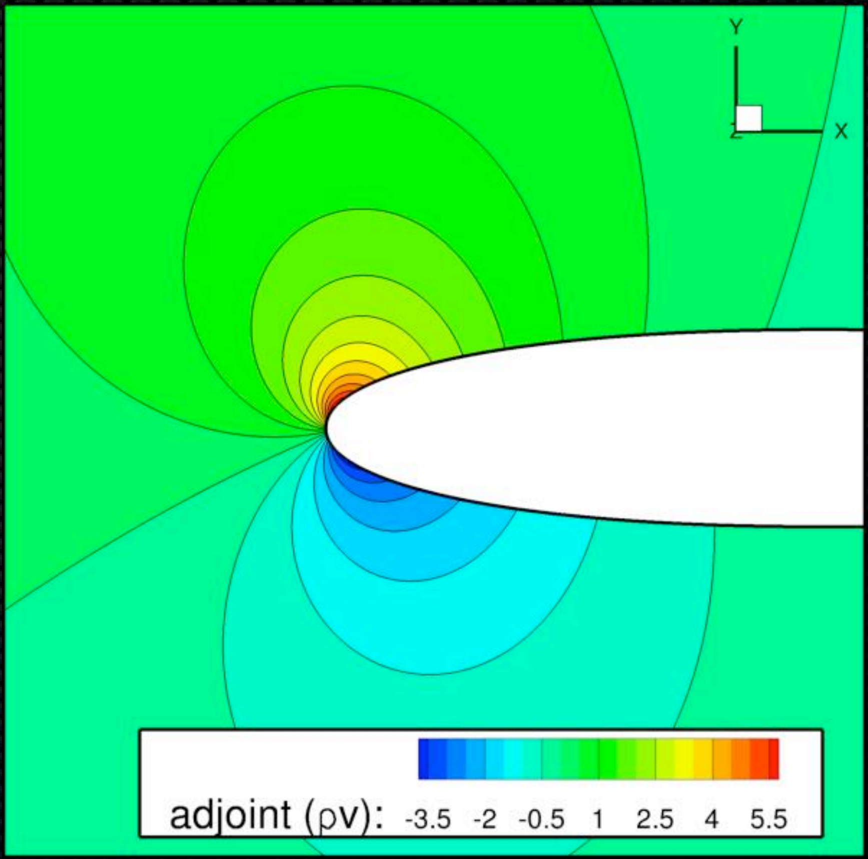
- A scheme is dual consistent if the associated discrete dual (or adjoint) problem is a consistent discretization of the continuous adjoint problem
  - ➡ Dual consistency requires suitable boundary conditions and a particular numerical integration method for the functional
  - ➡ Can lead to superconvergence of functionals
  - ➡ Can lead to much better error estimates based on adjoint-weighted residuals (than dual inconsistent schemes)

➡ Hicken, J.E., and Zingg, D.W., The Role of Dual Consistency in Functional Accuracy: Error Estimation and Superconvergence, 20th AIAA CFD Conference, June 2011.

# Dual Consistency

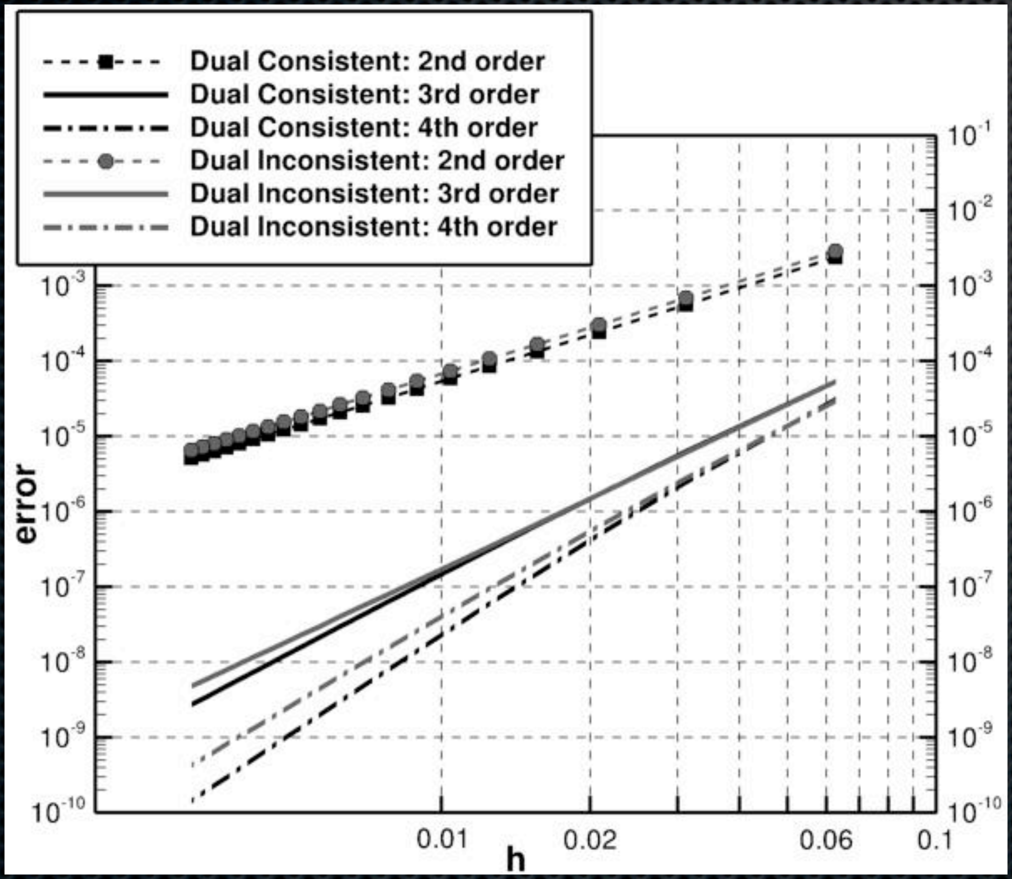
Example: adjoint field shows oscillations in dual inconsistent case



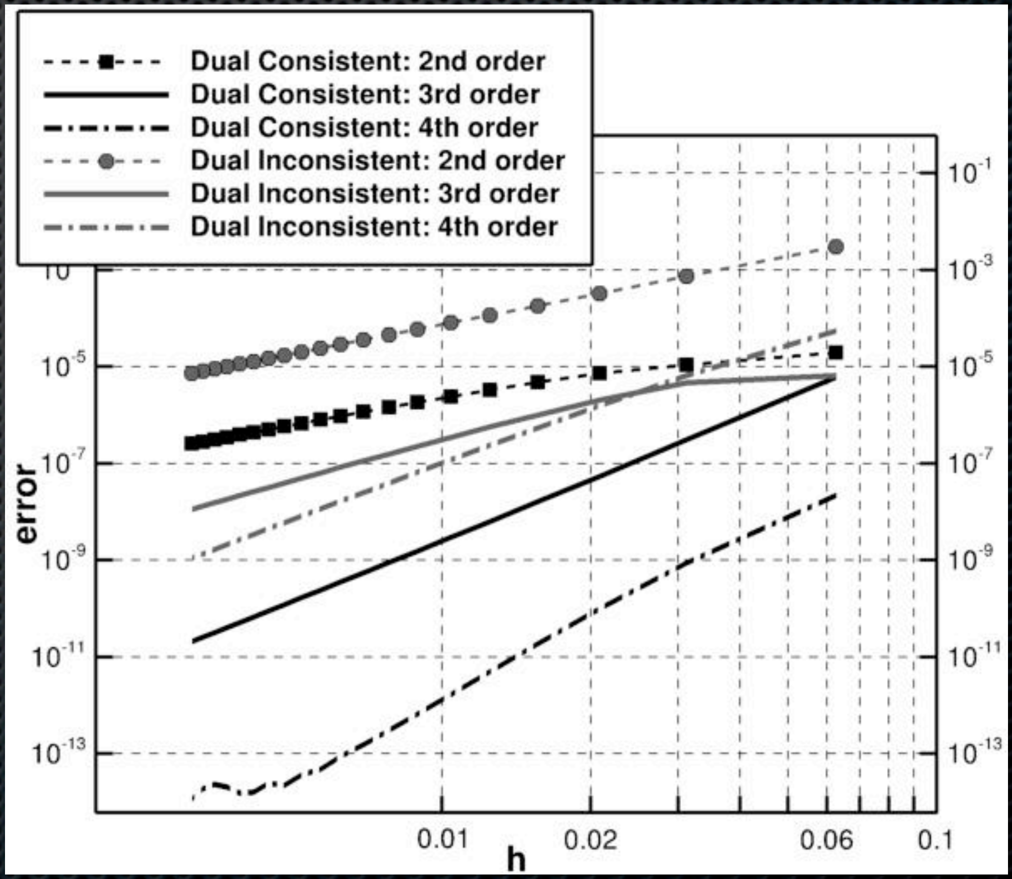


# Results for inviscid vortex flow

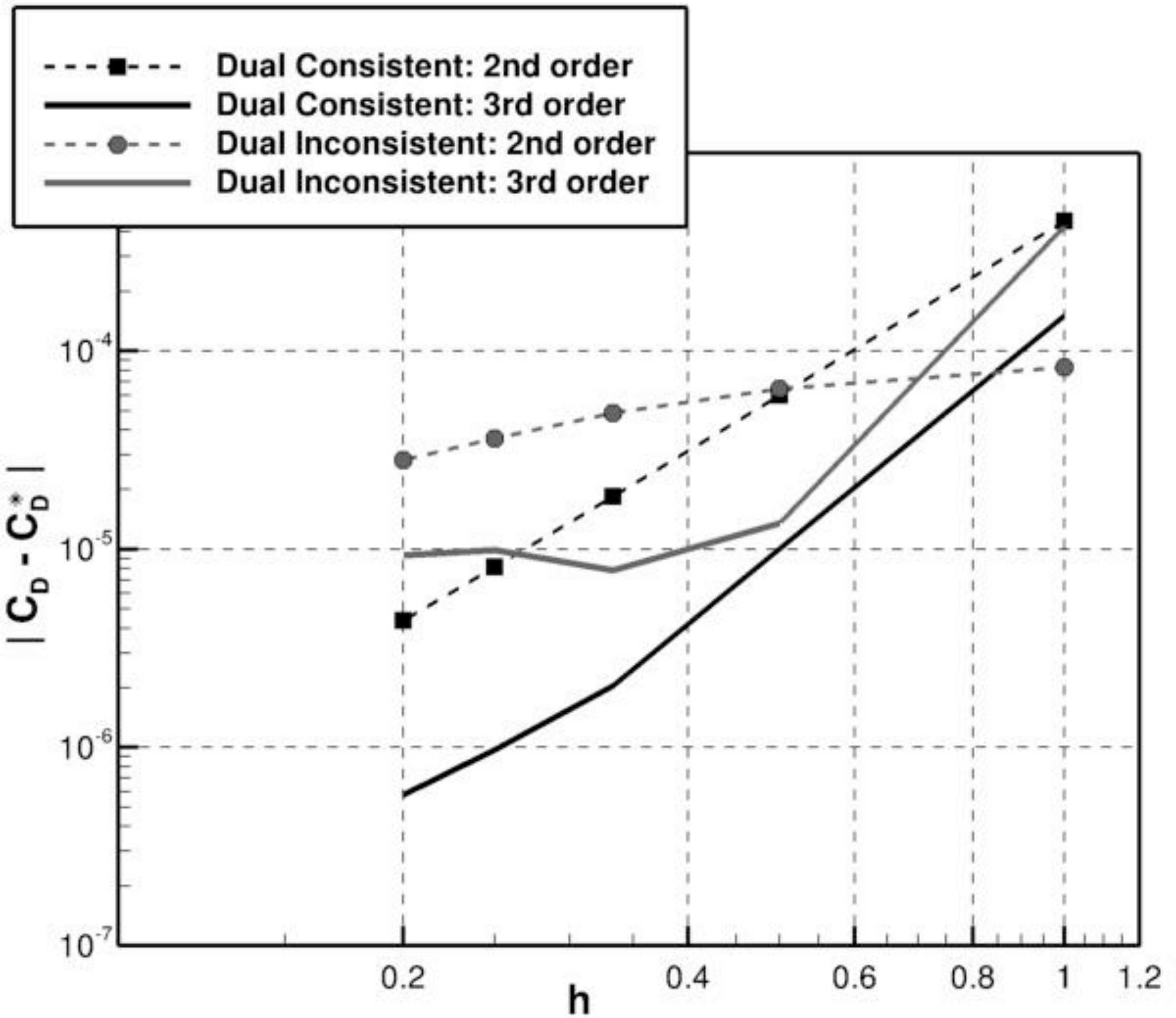
Solution error



Functional error



# Results for ONERA M6 wing



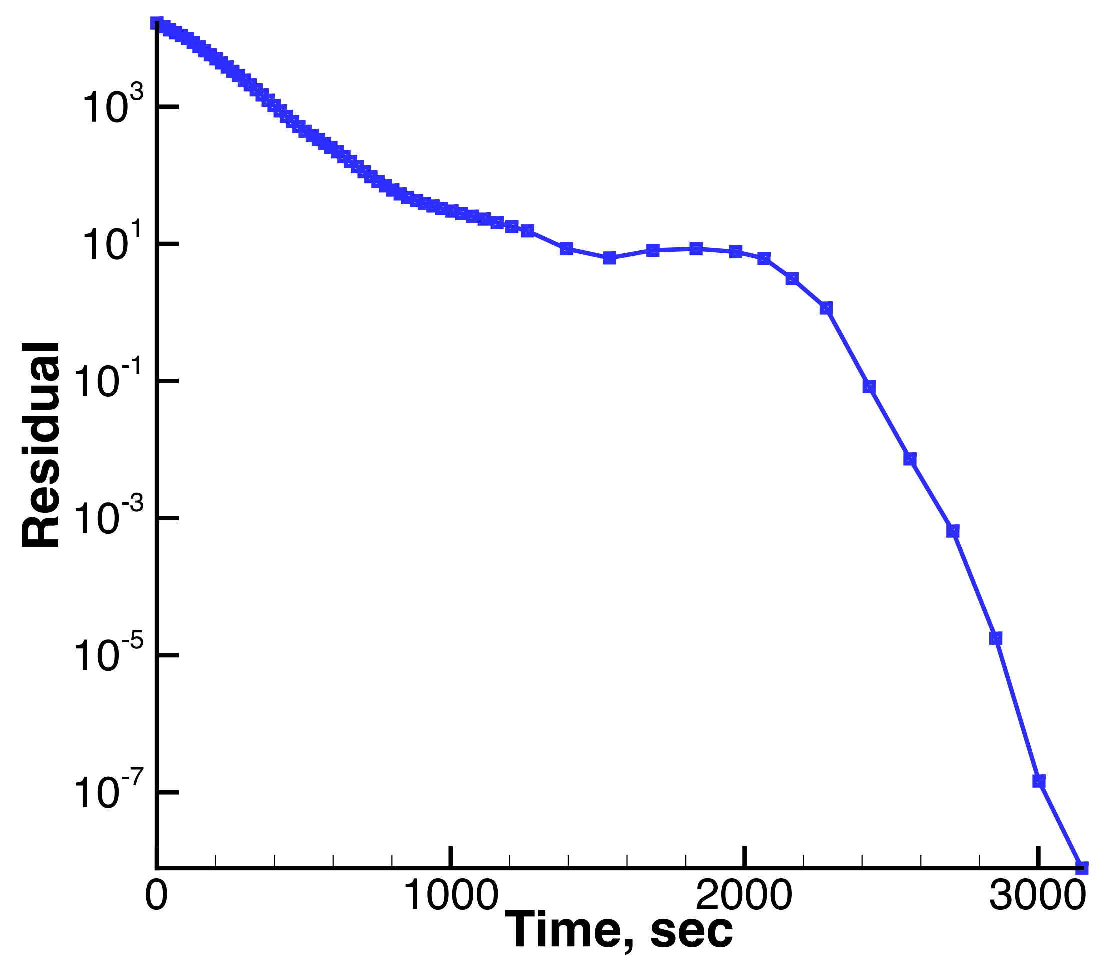
# FLOW SOLVER

- Structured multi-block grids
- High-order finite-difference method with summation-by-parts operators and simultaneous approximation terms
- Parallel Newton-Krylov-Schur solver
- Jacobian-free Newton-Krylov algorithm with approximate Schur parallel preconditioning
- Promising dissipation-based continuation method for globalization

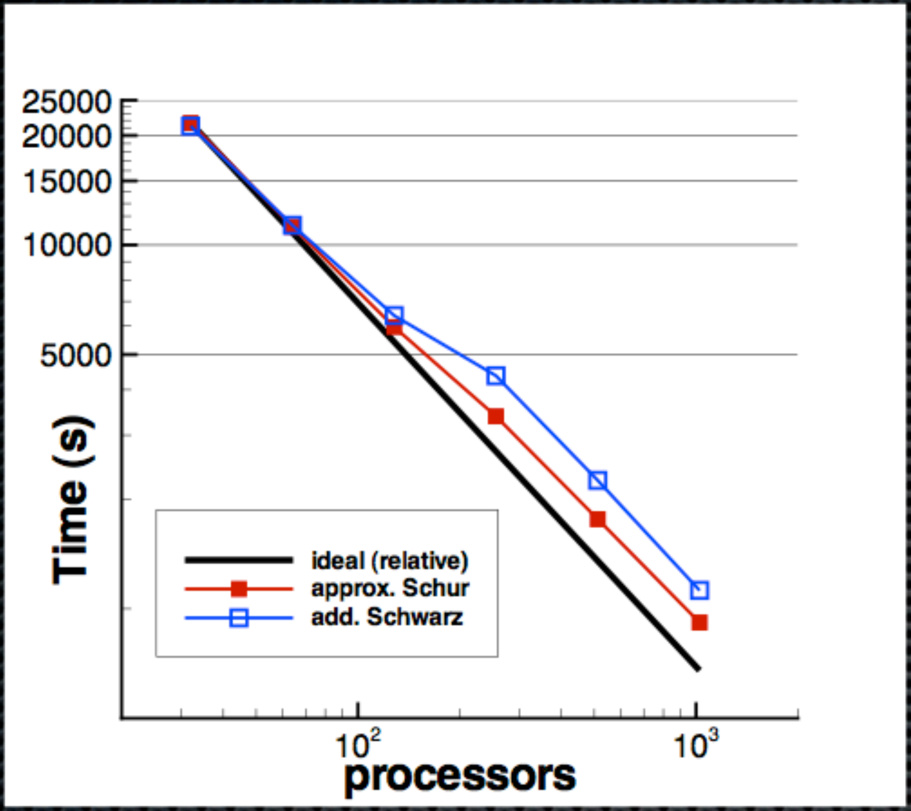
➡ Hicken, J.E., and Zingg, D.W., A parallel Newton-Krylov solver for the Euler equations discretized using simultaneous approximation terms, AIAA Journal, Vol. 46, No. 11, 2008

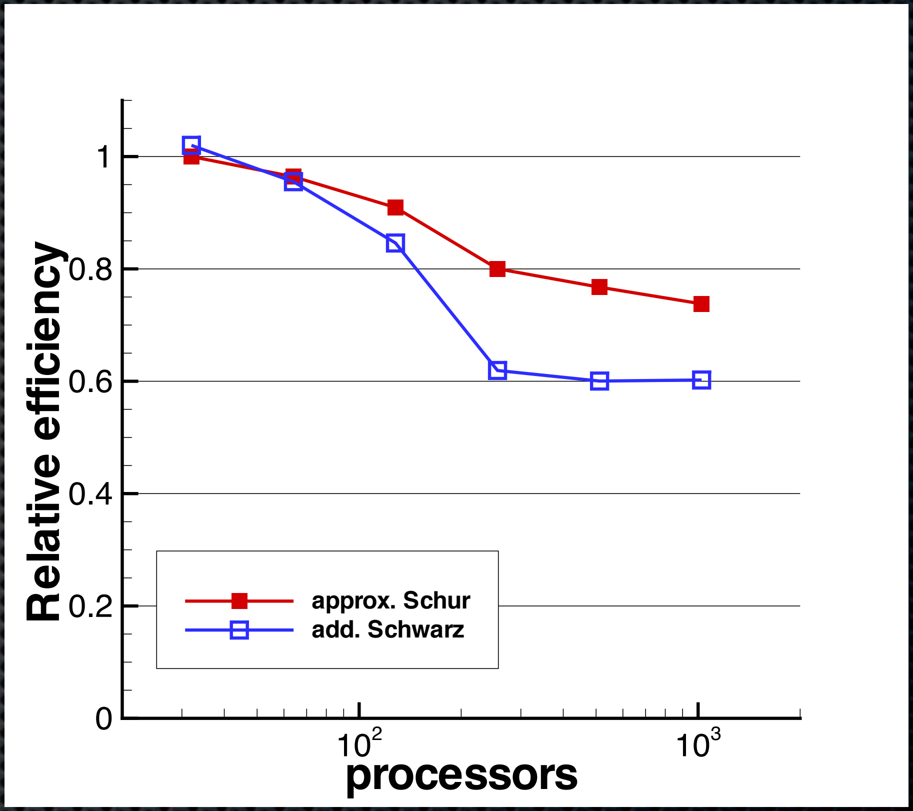
# Turbulent Flow Solver

ONERA M6 wing: M=0.8395, alpha=3.06 degrees
Re=11.72 million, 1.88 million mesh nodes, 16 processors



# Parallel Scalability (Euler)





- mesh with 38 million nodes
- 9-order residual reduction in 15 minutes on 1024 processors

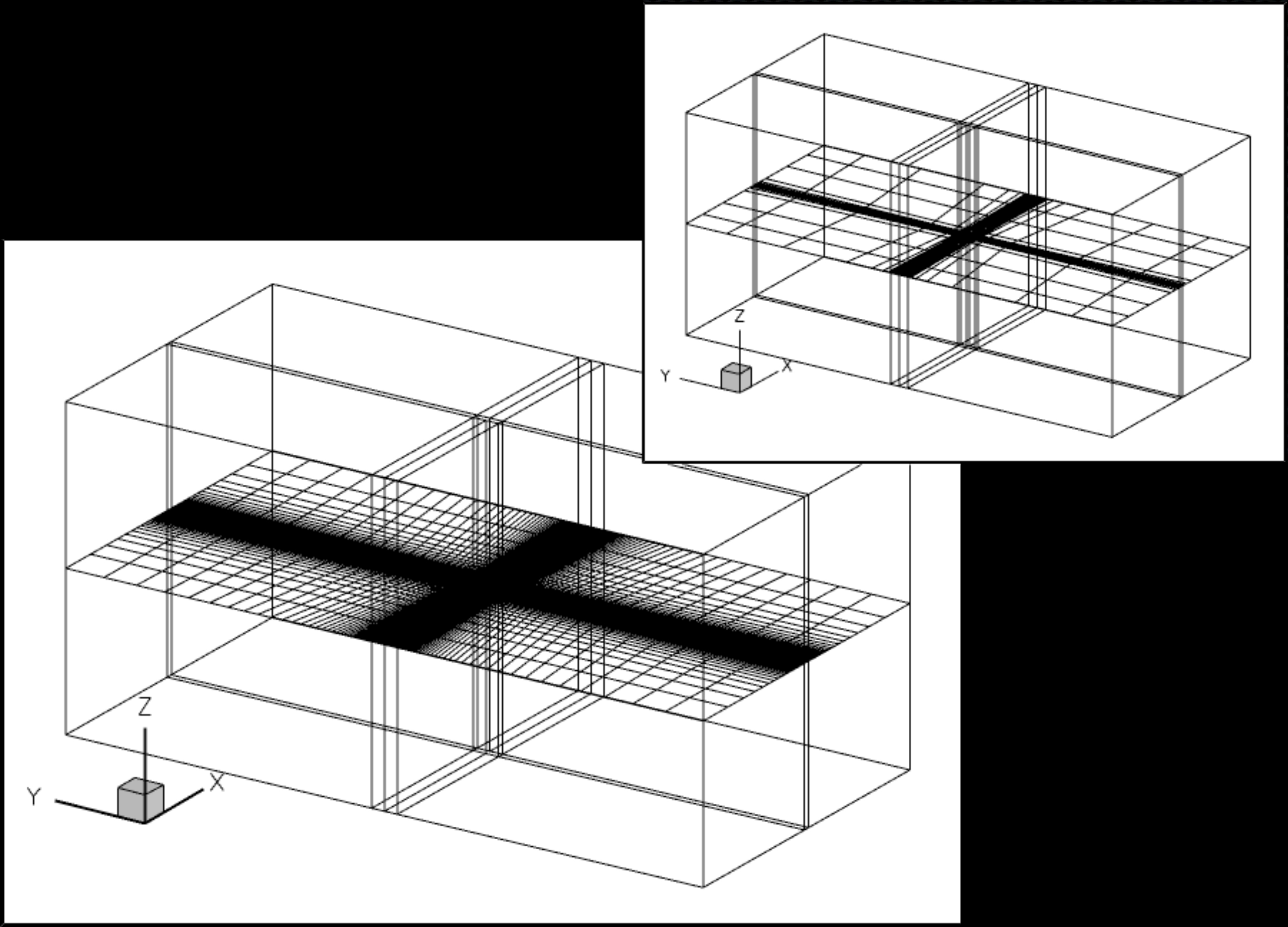
# INTEGRATED GEOMETRY PARAMETERIZATION AND MESH MOVEMENT

- Must provide flexibility for large shape changes with a modest number of design variables
    - B-spline patches represent surfaces
    - B-spline control points are design variables
- Mesh movement must maintain quality through large shape changes
    - through tensor products, B-spline volumes map a cube to an arbitrary volume with the appropriate topology
    - can be arbitrarily discretized in the cube domain to create a mesh
    - B-spline volume control points can be manipulated to move the mesh in response to changes in the surface control points
    - efficiently generates a high quality mesh

➡ Hicken, J.E., and Zingg, D.W., Aerodynamic Optimization Algorithm with Integrated Geometry Parameterization and Mesh Movement, AIAA Journal, Vol. 48, No. 2, 2010

# Mesh Movement Example

## flat plate to blended-wing body: ≈ 1 million nodes



# DISCRETE-ADJOINT GRADIENT COMPUTATION

- Cost independent of the number of design variables
- Efficient if the number of design variables exceeds the number of constraints
- Hand linearization complemented by judicious use of the complex step method for difficult terms
- Adjoint equation solved by parallel Schur-preconditioned modified Krylov method GCROT(m,k)

➡ Hicken, J.E., and Zingg, D.W., A Simplified and Flexible Variant of GCROT for Solving Nonsymmetric Linear Systems, SIAM Journal on Scientific Computing, Vol. 32, No. 3, March 2010

# Design Problem Definition

- Aerodynamic design specification for a hypothetical aircraft:
  - Cruise Mach number range: 0.78 - 0.88
  - Cruise weight range: 60,000 - 100,000 lbs
  - Cruise altitude range: 29,000 - 39,000 ft
  - Target airfoil thickness to chord ratio: 11.8%
  - On-design operating conditions: cruise and long-range cruise
  - Off-design operating conditions: dive conditions, low-speed conditions
  - Wing area: 1000 sq.ft.
  - Wing sweep angle: 35 degrees

# Weighted Integral Objective Function

- Design objective: maximize *L/D* over a range of cruise operating conditions
- Optimal solution minimizes the integral of *D/L* over a range of Mach numbers, aircraft weights, and altitudes
- A weighting function $\mathcal{D}$ is used to prioritize operating conditions
- The weighted integral is defined as:

$$\int_{h_1}^{h_2}\int_{W_1}^{W_2}\int_{M_1}^{M_2}\frac{D}{L}\,(M,W,h)\,\mathcal{D}\,(M,W,h)\;dM\,dW\,dh$$

# Designer Priority Weighting Function

- A sample weighting function is applied to test cases to illustrate the weighted integral approach
- Assume a constant cruise altitude…flight envelope is represented by a 2D integral:

$$\boxed{\int_{W_1}^{W_2}\int_{M_1}^{M_2}\frac{D}{L}(M,W)\,\mathcal{D}(M)\,dM\,dW}$$

$$\boxed{\mathcal{D}(M)=e^{a(M-M_1)}}$$

$$\boxed{a=\frac{\ln(20)}{M_2-M_1}}$$

- $\mathcal{D}(M_1) = 1, \mathcal{D}(M_2) = 20$ for $M_1 < M_2$
- Compare with cases where equal priority is given to all of the operating conditions; i.e. $\mathcal{D} = 1$

# Integral Approximation

- Objective function is defined as an approximation of the weighted integral

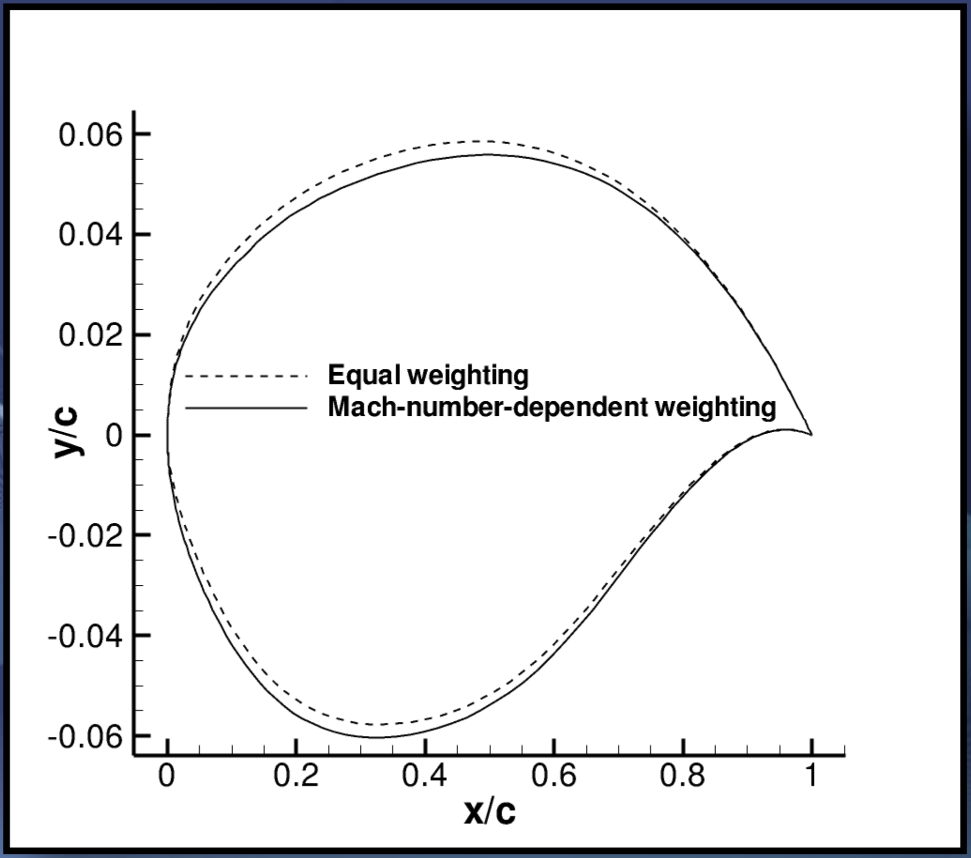
$$\mathcal{J} = \sum_{i=1}^{N_M} \sum_{j=1}^{N_W} \mathcal{T}_{i,j} \frac{D}{L} \left(M_i, W_j\right) \mathcal{D} \left(M_i, W_j\right) \Delta M \Delta W \simeq \int_{W_1}^{W_2} \int_{M_1}^{M_2} \frac{D}{L} \left(M, W\right) \mathcal{D} \left(M, W\right) dM dW$$

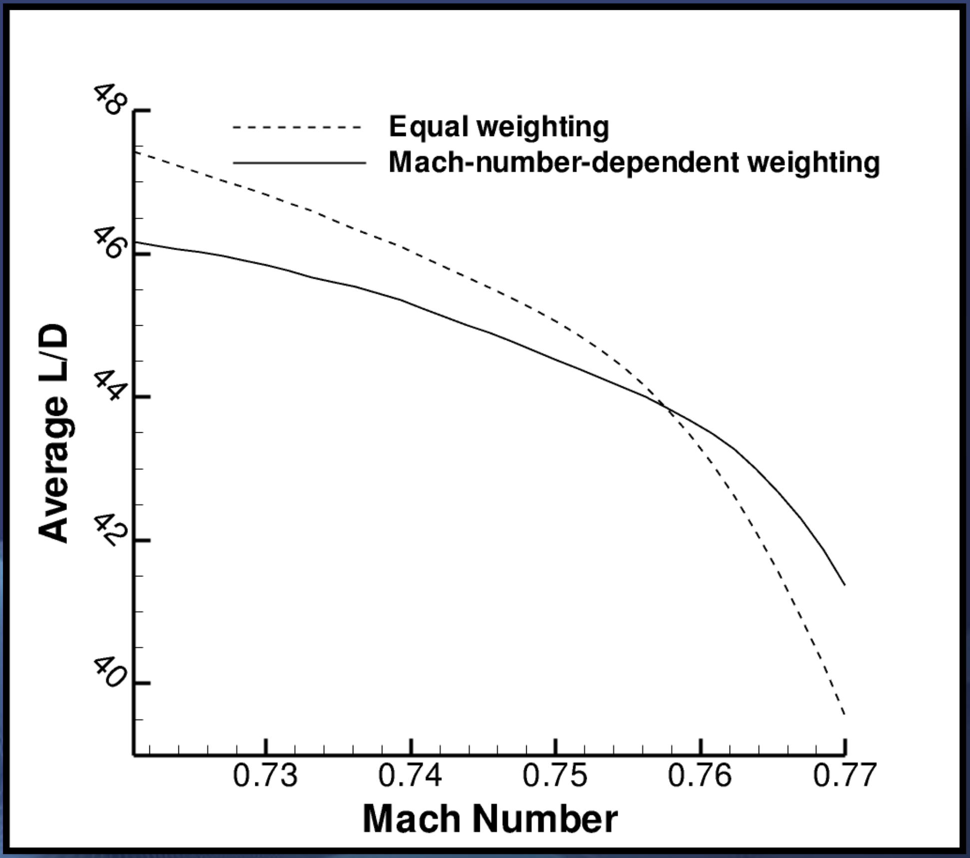
- $N_M$ x $N_W$ is the number of quadrature points used in $M$, $W$
- $\mathcal{T}_{i,j}$ are the weights used to approximate the integral using the trapezoidal quadrature rule

# Optimization Setup Parameters For Test Cases

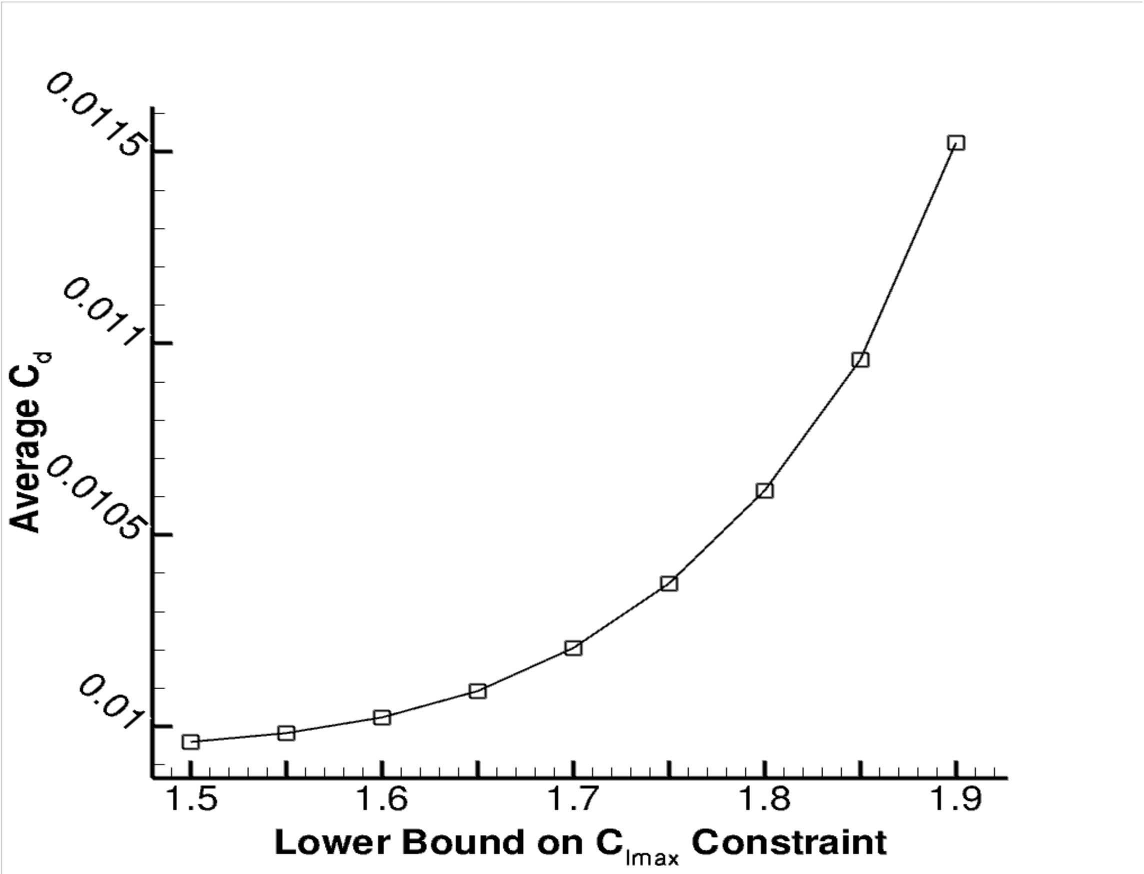
- Initial airfoil geometry: RAE 2822
- Geometry parameterization:
  - 15 B-spline control points
  - 12 design variables
- Mesh parameters:
  - C topology
  - 18785 nodes
  - Off-wall spacing = 2 x 10e-6
- Off-design constraints:
  - At dive conditions: $M_{\text{max}} \leq 1.35$
  - At low-speed conditions: $C_{\text{l,max}} \geq 1.60$
- Geometric constraints:
  - 2 thickness constraints at 95% and 99% chord
  - Area constraint

# Comparison of Equal Weighting vs. Mach-Number-Dependent Weighting





# Trade offs



Pareto front showing trade-off between cruise condition drag performance and $C_{lmax}$ constraint at low-speed conditions.

# Problem formulation

- demonstrated an effective approach to formulating design problems as optimization problems
- however, an aircraft has an enormous number of configurations, maneuvers, and cases that must be included
- some thought must be given to determining the minimum number of operating conditions that need to be considered

# Genetic algorithm or gradient-based adjoint method?

- Global optimization algorithms, e.g. genetic algorithms, are generally slow
- Gradient-based algorithms converge to a local minimum
- Preference depends on multimodality, among other considerations
- Yet there are virtually no studies of multimodality in aerodynamic shape optimization

➡ Chernukhin, O., and Zingg, D.W., An Investigation of Multi-Modality in Aerodynamic Shape Optimization, 20th AIAA Computational Fluid Dynamics Conf, June 2011

# Multimodality questions

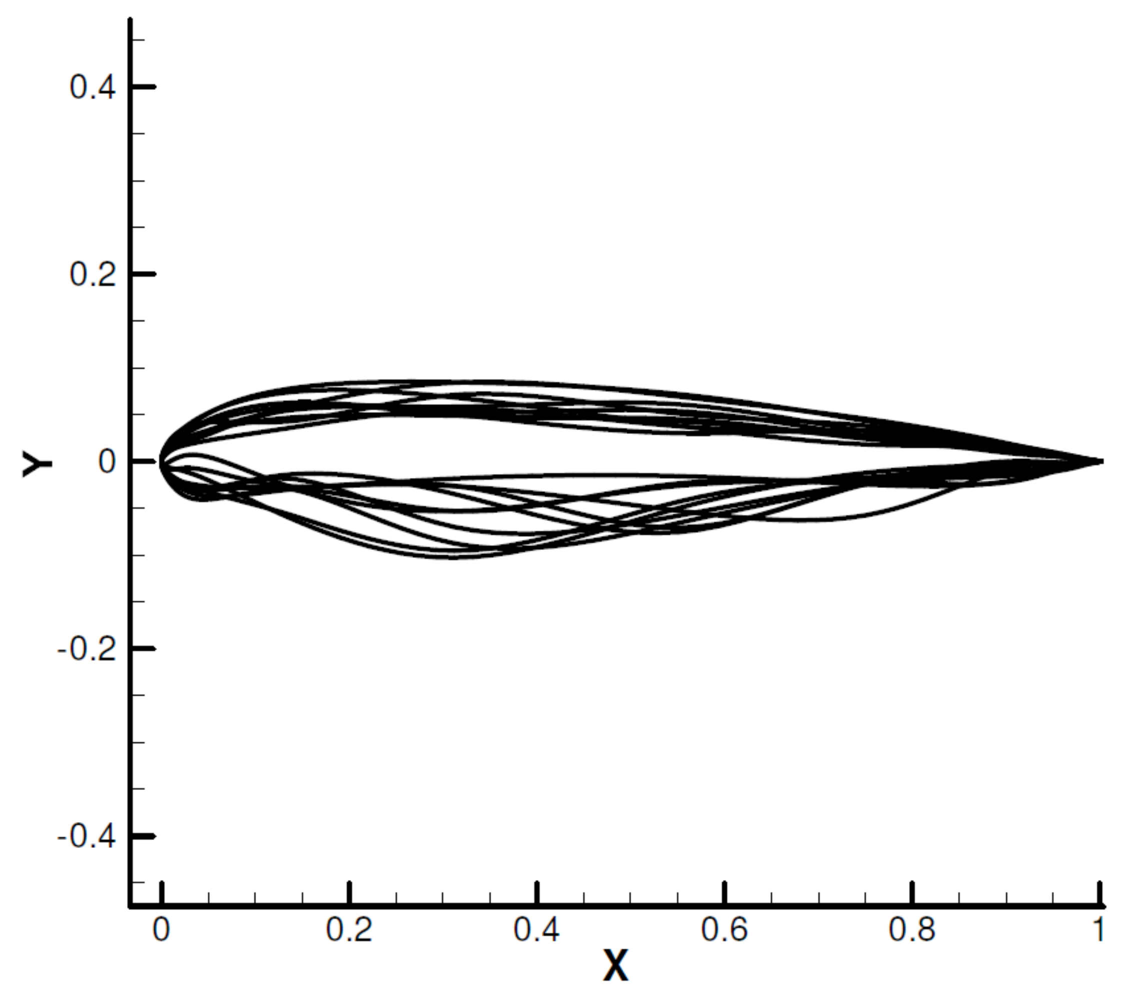
- Are our design spaces multimodal?
- If so, are they highly multimodal, moderately multimodal, somewhat multimodal, or unimodal?
- For each category, what is the best optimization algorithm for finding the global minimum?

# Four Optimization Algorithms

- Gradient-based algorithm (GB)
- Multi-start Sobol (GB-MS): initial guesses based on Sobol sequences cover the design space in a deterministic manner (sampling in linear feasible region)
- Hybrid method (HM): combination of genetic algorithm, Sobol sampling, and gradient-based algorithm (SNOPT is run on each chromosome)
- Genetic algorithm (GA)

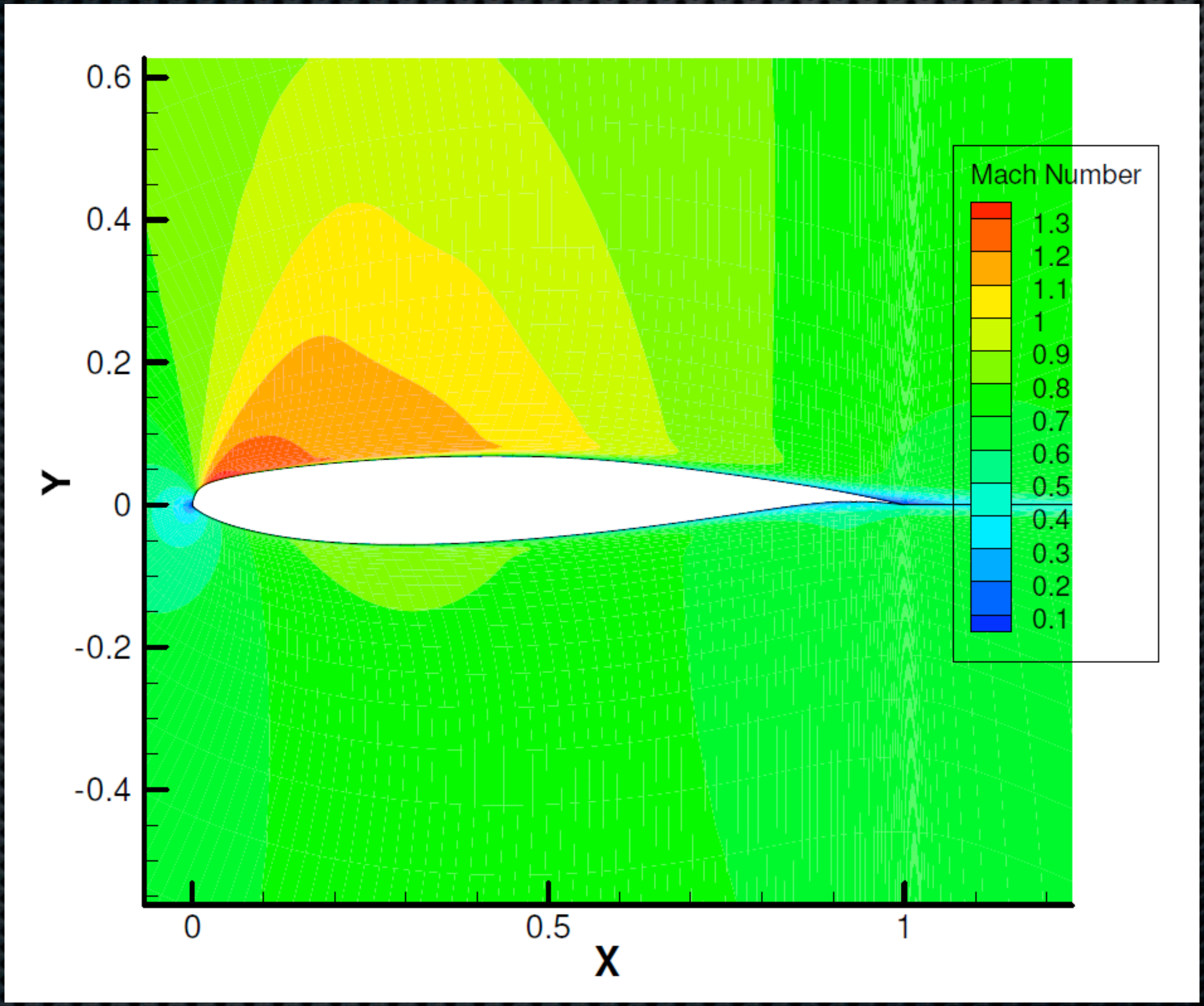
# Multimodality in 2D (RANS)

Multistart procedure for 2D airfoil optimization (transonic lift-constrained drag minimization, 6 DVs)



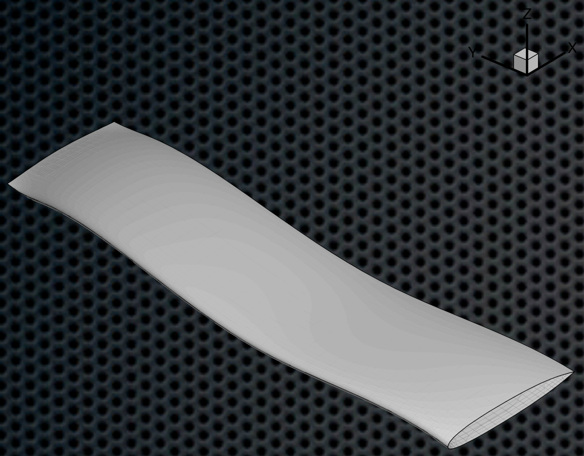
# Multimodality?

A unique global optimum in 2D - no local optima!



# Multimodality in 3D (Euler)

- transonic lift-constrained drag minimization, 129 DVs
- 3 local minima found - somewhat multimodal



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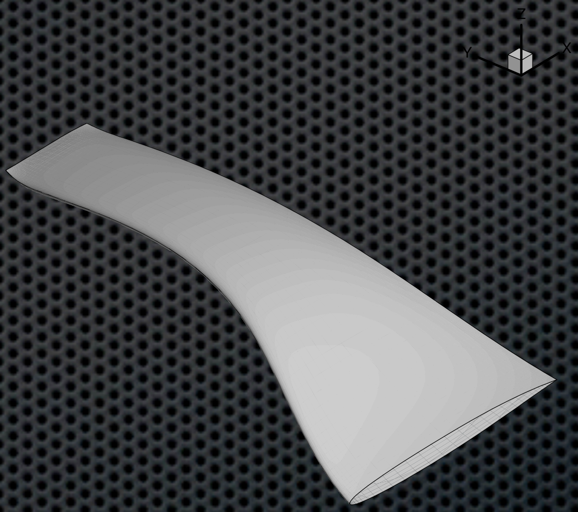

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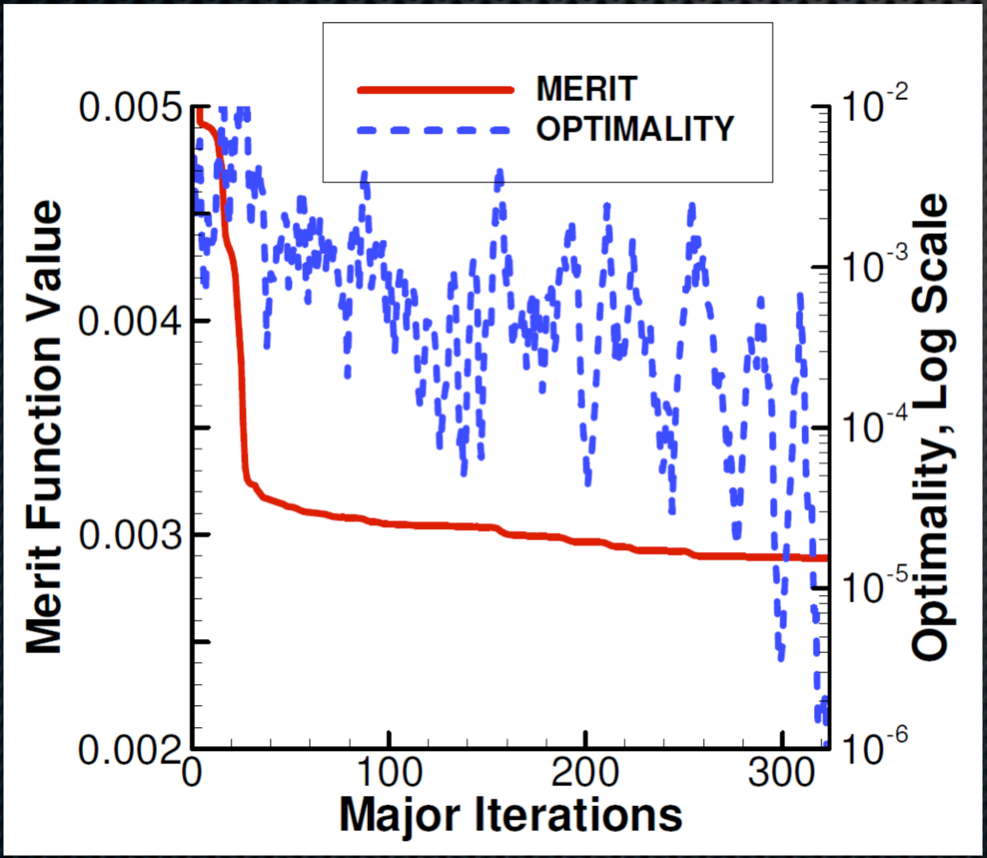
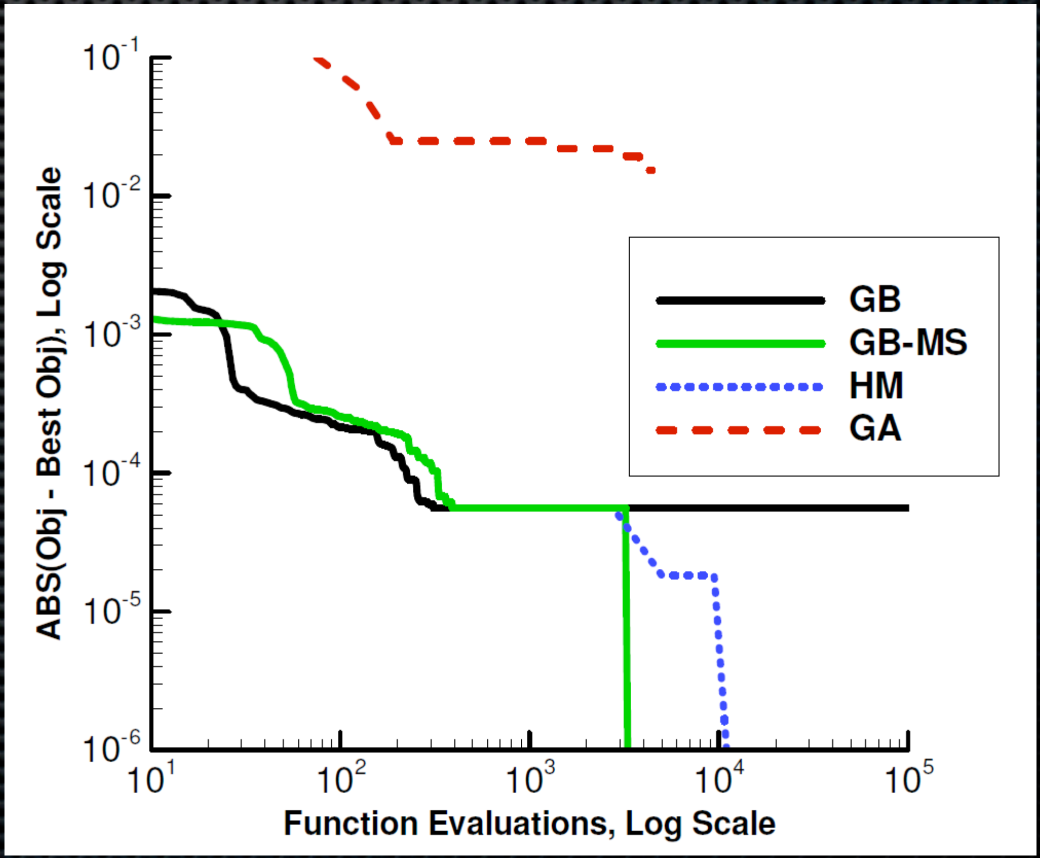
# Convergence to global minimum

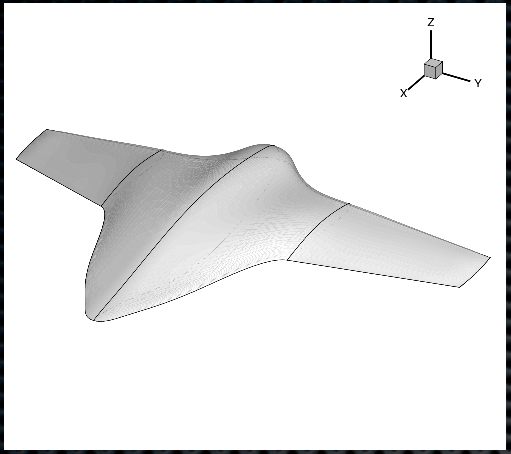
## Gradient-based algorithm

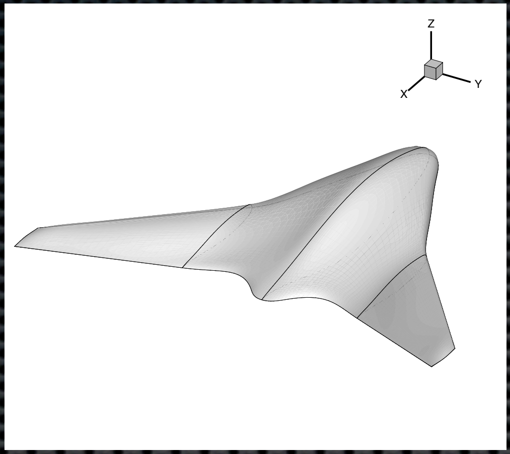


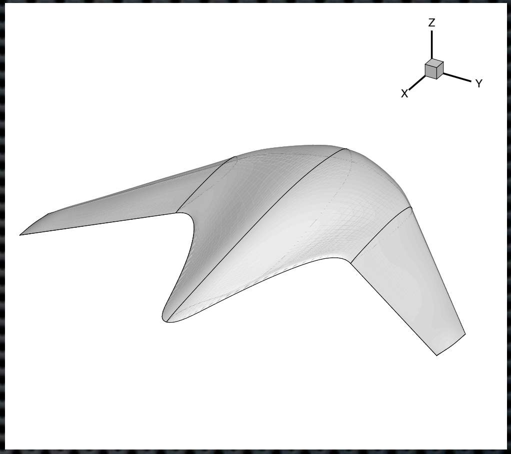
## All algorithms

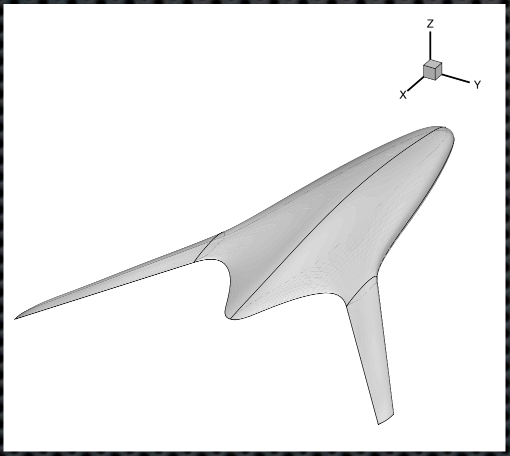


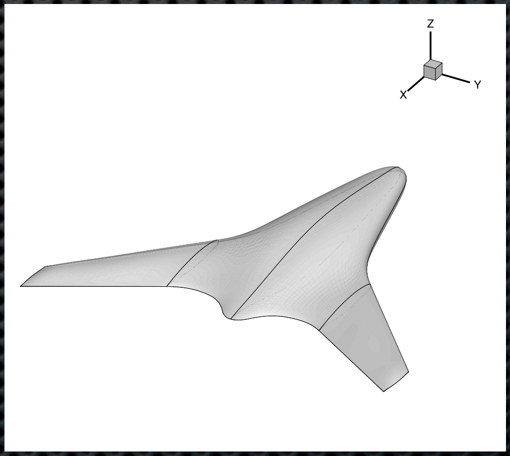
# Hybrid wing-body optimization











- 16 initial geometries ... 5 local optima ...

# What can we conclude?

- 2D RANS airfoil optimization appears to be unimodal
  - gradient-based algorithm is suitable
- 3D Euler wing optimization somewhat multimodal depending on degree of geometric flexibility
  - gradient-based multi-start algorithm is preferred
- Hybrid wing-body optimization has a higher degree of multimodality presumably because of its high degree of geometric flexibility
  - global optimization algorithm (but not a GA) preferred for exploratory optimization
  - multi-start gradient-based algorithm based on Sobol sequence a good place to start

# Future Work

- higher-order dual consistent SBP operators for viscous terms
- laminar-turbulent transition in optimization
- aerostructural optimization
- strategies for improving efficiency
- strategies for improving automation
- applications
    - unconventional configurations: development and evaluation
    - both incremental and exploratory - what can we discover?
    - flow control design through optimization (unsteady)