FINAL TEST, DECEMBER 2002
AER 1316H - FUNDAMENTALS OF CFD
120 minutes

1. Using a Taylor table, derive a fourth-order compact finite-difference operator for a second derivative, that is derive Eq. 3.52 in the text. (20 marks)

2. Consider the following time-marching method:

$$\begin{aligned} \tilde{u}_{n+1/3} &= u_n + hu'_n/3 \\ \bar{u}_{n+1/2} &= u_n + h\tilde{u}'_{n+1/3}/2 \\ u_{n+1} &= u_n + h\bar{u}'_{n+1/2} \end{aligned}$$

Find the difference equation which results from applying this method to the representative equation. Find the λ–σ relation. Find the solution to the difference equation, including the homogeneous and particular solutions. Find er_λ and er_μ. What order is the homogeneous solution? What order is the particular solution? Find the particular solution if the forcing term is fixed. (40 marks)

3. Consider solving the diffusion equation with Dirichlet boundary conditions using the compact spatial operator given in Eq. 3.52. Write the system of ODE's for the semi-discrete form. Find the λ eigenvalues. (20 marks)

4. Repeat the time-march comparison for the periodic convection problem given in the textbook using the third-order Runge-Kutta method. Find the quantities shown in Table 8.2. Note that you need not derive the λ-σ relation for the third-order Runge-Kutta method; you can deduce it from Eq. 6.69. (20 marks)