FINAL TEST
AER 1316H - FUNDAMENTALS OF CFD
120 minutes

1. Consider the fourth-order noncompact finite-difference approximation to a first derivative given in (3.54). Using this operator to approximate the spatial derivative in the linear convection equation, write the semi-discrete form obtained with periodic boundary conditions on a 5-point grid ($M = 5$). Write out all matrices and vectors in full. (15 marks)

2. The 2nd-order backward method is given by

$$u_{n+1} = \frac{1}{3}\left(4u_n - u_{n-1} + 2hu'_{n+1}\right) \ .$$

(a) Write the OΔE for the representative equation. Identify the polynomials $P(E)$ and $Q(E)$.

(b) Derive the λ–σ relation. Solve for the σ-roots and identify them as principal or spurious.

(c) Find er_λ.

(d) Find the particular solution if the forcing term is fixed ($\mu = 0$).

(25 marks)

3. Consider the following time-marching method:

$$\begin{aligned}
\tilde{u}_{n+1/3} &= u_n + hu'_n/3 \\
\bar{u}_{n+1/2} &= u_n + h\tilde{u}'_{n+1/3}/2 \\
u_{n+1} &= u_n + h\bar{u}'_{n+1/2}
\end{aligned}$$

Find the difference equation which results from applying this method to the representative equation. Find the λ–σ relation. Find the solution to the difference equation, including the homogeneous and particular solutions. Find er_λ and the c_0 term in the er_μ calculation. What order is the homogeneous solution? Find the particular solution if the forcing term is fixed. (25 marks)

4. Using Fourier analysis, analyze the stability of first-order backward differencing coupled with explicit Euler time marching applied to the linear convection equation with positive a. Find the maximum Courant number for stability. (15 marks)

5. A second-order backward difference approximation to a first derivative is given as a point operator by

$$(\delta_x u)_j = \frac{1}{2\Delta x}(u_{j-2} - 4u_{j-1} + 3u_j) \ .$$

(a) Express this operator in banded matrix form (for periodic boundary conditions), then derive the symmetric and skew-symmetric matrices that have the matrix operator as their sum. (See Appendix A.3 to see how to construct the symmetric and skew-symmetric components of a matrix.) (10 marks)

(b) Using a Taylor table, find the derivative which is approximated by the corresponding symmetric and skew-symmetric operators and the leading error term for each. (10 marks)