FINAL TEST
AER 1316H - FUNDAMENTALS OF CFD
120 minutes

1. A time-marching method is *L-stable* if it is A-stable and $\lim_{|\lambda h|\to\infty} \sigma(\lambda h) = 0$. Which of the following methods is L-stable: implicit Euler, trapezoidal, second-order backwards, leapfrog, Gazdag? (5 marks)

2. Apply Fourier stability analysis to the full discretization of the diffusion equation obtained from the combination of second-order centered differencing in space

$$(\delta_{xx}u)_j = \frac{u_{j+1} - 2u_j + u_{j-1}}{\Delta x^2}$$

and the second-order Adams-Bashforth method. Assume that the critical case occurs when $\kappa\Delta x = \pi$, and show that the von Neumann number, $\nu h/(\Delta x^2)$, cannot exceed 1/4 for stability. (15 marks)

3. Write the semi-discrete form of the diffusion equation with periodic boundary conditions obtained from second-order centered differencing in space. Find the λ-eigenvalues of the semi-discrete operator matrix. Using the $\lambda - \sigma$ relation for the second-order Adams-Bashforth method given on page 122 of the text, evaluate σ for $m = M/2$ and a von Neumann number of 1/4. Which value of σ corresponds to the principal root? (20 marks)

4. Find the $\lambda - \sigma$ relation for the two-step linear multi-step method obtained from Eq. 6.59 with $\theta = 1/3$, $\xi = -1/6$, $\phi = 0$. Find the σ-roots for $\lambda h = 0$. What order is this method? Is it explicit or implicit? (20 marks)

5. (a) Using a Taylor table, Derive a finite-difference approximation to a first derivative in the form

$$a(\delta_x u)_{j-1} + (\delta_x u)_j = \frac{1}{\Delta x}(bu_{j-1} + cu_j + du_{j+1}) \ .$$

(b) Find the leading error term. Is the leading error term dissipative or dispersive? Why? (20 marks)

6. Using Appendix B.2, find the eigenvalues of $H^{-1}A$ for the SOR method with $A = B(4 : 1, -2, 1)$ and $\omega = \omega_{\text{opt}}$. (You do not have to find H^{-1}. Recall that the eigenvalues of $H^{-1}A$ satisfy $A\vec{x}_m = \lambda_m H\vec{x}_m$.) Find the numerical values, not just the expressions. Then find the corresponding $|\sigma_m|$ values. (20 marks)