AERODYNAMICS PROBLEM SET 2 SOLUTIONS

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Problem 1

L=73600N, b=15.23m, and v=90 m/s

$$\begin{aligned} q_\infty &= \frac{1}{2}\rho_\infty v_\infty^2 \\ &= \frac{1}{2}\left(1.226\frac{kg}{m^3}\right)\left(90\frac{m}{s}\right) \\ &= 4965.3\frac{kg}{ms^2} \end{aligned}$$

The coefficient of drag is given by

$$C_{D,i} = \frac{C_L^2}{\pi AR}(1+\delta) \tag{1}$$

For elliptical wings, $\delta = 0$
So, the induced drag for this wing would be,

$$D_i = \frac{C_L^2}{\pi AR} q_\infty S \tag{2}$$

The coefficient of lift is given by

$$C_L = \frac{L}{q_\infty S} \tag{3}$$

Substituting Eq. 3 into Eq. 2 for C_L gives

$$D_i = \frac{L^2}{q_\infty S \pi AR} \tag{4}$$

but $AR = \frac{b^2}{S}$
so

$$D_i = \frac{L^2}{q_\infty \pi b^2}$$

$$
\begin{aligned}
&= \frac{(73600N)^2}{\pi\left(4965.3\frac{kg}{ms^2}\right)(15.23m)^2} \\
&= 1480N
\end{aligned}
$$

Problem 2

for a general wing,

$$\alpha = \frac{2b}{\pi c(\theta_0)} \sum_1^N A_n \sin n\theta_0 + \alpha_{L=0} + \sum_1^N nA_n \frac{\sin n\theta_0}{\sin \theta_0}$$

Chose $\theta_0 = \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}, \frac{5\pi}{8}, \frac{3\pi}{4}, \frac{7\pi}{8}$
The corresponding chord lengths are $c(\theta_0) = 0.446c, 0.576c, 0.770c, c, c, 0.770c, 0.576c, 0.446c$
(where c is the chord length at the root of the wing)
Each of these values of θ gives one equation with 7 unknowns (A_n)
for $\theta_0 = \frac{\pi}{8}$ we have

$$
\begin{aligned}
\alpha &= \frac{2b}{\pi 0.446c}\left[A_1 \sin\frac{\pi}{8} + A_2 \sin\frac{\pi}{4} + A_3 \sin\frac{3\pi}{8} + A_4 \sin\frac{\pi}{2} + A_5 \sin\frac{5\pi}{8} + A_6 \sin\frac{3\pi}{4} + A_7 \sin\frac{7\pi}{8}\right] \\
&\quad -\alpha_{L=0} \\
&\quad + \left[A_1 + 2A_2\frac{\sin\frac{\pi}{4}}{\sin\frac{\pi}{8}} + 3A_3\frac{\sin\frac{3\pi}{8}}{\sin\frac{\pi}{8}} + 4A_4\frac{\sin\frac{\pi}{2}}{\sin\frac{\pi}{8}} + 5A_5\frac{\sin\frac{5\pi}{8}}{\sin\frac{\pi}{8}} + 6A_6\frac{\sin\frac{3\pi}{4}}{\sin\frac{\pi}{8}} + 7A_7\frac{\sin\frac{7\pi}{8}}{\sin\frac{\pi}{8}}\right]
\end{aligned}
$$

similarly, for $\theta_0 = \frac{7\pi}{8}$ we have

$$
\begin{aligned}
\alpha &= \frac{2b}{\pi 0.446c}\left[A_1 \sin\frac{7\pi}{8} + A_2 \sin\frac{7\pi}{4} + A_3 \sin\frac{21\pi}{8} + A_4 \sin\frac{7\pi}{2} + A_5 \sin\frac{35\pi}{8} + A_6 \sin\frac{21\pi}{4} + A_7 \sin\frac{49\pi}{8}\right] \\
&\quad -\alpha_{L=0} \\
&\quad + \left[A_1 + 2A_2\frac{\sin\frac{7\pi}{4}}{\sin\frac{7\pi}{8}} + 3A_3\frac{\sin\frac{21\pi}{8}}{\sin\frac{7\pi}{8}} + 4A_4\frac{\sin\frac{7\pi}{2}}{\sin\frac{7\pi}{8}} + 5A_5\frac{\sin\frac{35\pi}{8}}{\sin\frac{7\pi}{8}} + 6A_6\frac{\sin\frac{21\pi}{4}}{\sin\frac{7\pi}{8}} + 7A_7\frac{\sin\frac{49\pi}{8}}{\sin\frac{7\pi}{8}}\right]
\end{aligned}
$$

Adding these two equation and dividing by 2 eliminates A_2, A_4, and A_6

$$
\begin{aligned}
\alpha &= \frac{2b}{\pi 0.446c}\left[A_1 \sin\frac{\pi}{8} + A_3 \sin\frac{3\pi}{8} + A_5 \sin\frac{5\pi}{8} + A_7 \sin\frac{7\pi}{8}\right] - \alpha_{L=0} \\
&\quad + \left[A_1 + 3A_3\frac{\sin\frac{3\pi}{8}}{\sin\frac{\pi}{8}} + 5A_5\frac{\sin\frac{5\pi}{8}}{\sin\frac{\pi}{8}} + 7A_7\frac{\sin\frac{7\pi}{8}}{\sin\frac{\pi}{8}}\right]
\end{aligned}
$$

Similarly, combining the equations from $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$, gives

$$
\begin{aligned}
\alpha &= \frac{2b}{\pi 0.576c}\left[A_1 \sin\frac{\pi}{4} + A_3 \sin\frac{3\pi}{4} + A_5 \sin\frac{5\pi}{4} + A_7 \sin\frac{7\pi}{4}\right] - \alpha_{L=0} \\
&\quad + \left[A_1 + 3A_3\frac{\sin\frac{3\pi}{4}}{\sin\frac{\pi}{4}} + 5A_5\frac{\sin\frac{5\pi}{4}}{\sin\frac{\pi}{4}} + 7A_7\frac{\sin\frac{7\pi}{4}}{\sin\frac{\pi}{4}}\right]
\end{aligned}
$$

And combining the equations from $\theta = \frac{3\pi}{8}, \frac{5\pi}{8}$, gives

$$\begin{aligned}\alpha &= \frac{2b}{\pi 0.770c}\left[A_1 \sin\frac{3\pi}{8} + A_3 \sin\frac{9\pi}{8} + A_5 \sin\frac{15\pi}{8} + A_7 \sin\frac{21\pi}{8}\right] - \alpha_{L=0} \\ &+ \left[A_1 + 3A_3\frac{\sin\frac{9\pi}{8}}{\sin\frac{3\pi}{8}} + 5A_5\frac{\sin\frac{15\pi}{8}}{\sin\frac{3\pi}{8}} + 7A_7\frac{\sin\frac{21\pi}{8}}{\sin\frac{3\pi}{8}}\right]\end{aligned}$$

Also, for $\theta = \frac{\pi}{2}$,

$$\begin{aligned}\alpha &= \frac{2b}{\pi c}\left[A_1 \sin\frac{\pi}{2} + A_3 \sin\frac{3\pi}{2} + A_5 \sin\frac{5\pi}{2} + A_7 \sin\frac{7\pi}{2}\right] - \alpha_{L=0} \\ &+ \left[A_1 + 3A_3\frac{\sin\frac{3\pi}{2}}{\sin\frac{\pi}{3}} + 5A_5\frac{\sin\frac{5\pi}{2}}{\sin\frac{\pi}{2}} + 7A_7\frac{\sin\frac{7\pi}{2}}{\sin\frac{\pi}{2}}\right]\end{aligned}$$

These equations can be put into matrix form Ax=b where

$$A = \begin{bmatrix} 4.4413 & 15.5507 & 20.3792 & 10.4413 \\ 5.9236 & 7.9236 & -9.9236 & -11.9236 \\ 5.8122 & -3.2359 & -4.0644 & 11.8122 \\ 5.0107 & -7.0107 & 9.0107 & -11.0107 \end{bmatrix}$$

$$x = \begin{bmatrix} A_1 \\ A_3 \\ A_5 \\ A_7 \end{bmatrix}$$

$$b = \begin{bmatrix} \alpha + \alpha_{L=0} \\ \alpha + \alpha_{L=0} \\ \alpha + \alpha_{L=0} \\ \alpha + \alpha_{L=0} \end{bmatrix}$$

Solving this system of equations gives

$$\begin{aligned} A_1 &= 0.1813(\alpha + \alpha_{L=0}) \\ A_3 &= 0.0008(\alpha + \alpha_{L=0}) \\ A_5 &= 0.0095(\alpha + \alpha_{L=0}) \\ A_7 &= -0.0011(\alpha + \alpha_{L=0}) \end{aligned}$$

Solving for A_2, A_4, A_6 will show that these are all equal to 0.
The coefficient of lift is

$$C_L = A_1 \pi AR = 5.126(\alpha + \alpha_{L=0})$$

And the coefficient of drag is

$$C_{D,i} = \frac{C_L^2}{\pi AR}(1 + \delta)$$

δ in the above expression is

$$\delta = \sum_{2}^{N} n \left(\frac{A_n}{A_1}\right)^2 = 0.014$$

So the expression for $C_{D,i}$ becomes

$$C_{D,i} = 0.94237(\alpha + \alpha_{L=0})^2$$

At an angle of attack of $4°$ (0.0698 rad), $C_L = 0.465$ and $C_{D,i} = 0.0078$

Problem 3

$$\theta = \nu(M_2) - \nu(M_1)$$

and

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)} - \tan^{-1}\sqrt{M^2-1}$$

Find $\nu(M_1 = 2.2)$

$$\nu(M_1) = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}(M_1^2-1)} - \tan^{-1}\sqrt{M_1^2-1} = 31.73$$

for $\theta = 4°$, $\nu(m_2) = 31.73 + 4 = 35.73$

$$35.73 = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1}(M_2^2-1)} - \tan^{-1}\sqrt{M_2^2-1}$$

Solving for M_2 in the above equation gives $M_2 = 2.3583$
Similarly, solving for M_2 with $\theta = 12°$ gives $M_2 = 2.7049$

Problem 4

$1.3atm = 131.7225kPa$

Bernoilli's Equation

Bernoulli's Equation:

$$p + \frac{1}{2}\rho v^2 = p_0 = const$$

$p_0 = 131.7225kPa$
$p = 101.325kPa$

So by applying Bernoulli's equation,

$$\begin{aligned}\frac{1}{2}\rho v^2 &= p_0 - p \\ v &= \sqrt{\frac{2(p_0-p)}{\rho}} \\ &= 222.7\frac{m}{s}\end{aligned}$$

Isentropic Relations

Applying the following isentropic relation

$$M^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

gives $M = 0.624$
which corresponds to a velocity of $v = 212\frac{m}{s}$

Problem 5

For $M_\infty = 0.4$ the flow should be considered compressible. At this freestream velocity, the point of minimum pressure on the airfoil has a pressure coefficient of $c_p = -0.782$
Assuming that the Prandtl-Glauert rule is sufficiently accurate, the pressure coefficient at this point for low freestream Mach numbers can be found

$$c_{p,0} = c_p\sqrt{1 - M_\infty^2} = -0.7167$$

So the point of minimum pressure on the airfoil will have a pressure given by

$$c_p = \frac{c_{p,0}}{\sqrt{1-M_\infty^2}} = \frac{-0.7167}{\sqrt{1-M_\infty^2}} \tag{5}$$

When the flow is at the critical Mach number (M_{cr}), the expression for the corresponding pressure coefficient at the point of minimum pressure is given by

$$C_{p_{cr}} = \frac{2}{\gamma M_{cr}^2}\left[\left(\frac{1+\frac{\gamma-1}{2}M_{cr}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma}{\gamma-1}} - 1\right] \tag{6}$$

Equations Eq. 5 and Eq. 6 can be solved to get the critical Mach number of the airfoil. $M_{cr} = 0.661$

Problem 6

We are given the following

$$\begin{aligned} p_1 &= 12054kPa \\ T_1 &= 216.5K \\ \rho_1 &= 0.194\frac{kg}{m^3} \\ M_1 &= 3.5 \end{aligned}$$

Using appendix A we have

$$p_{01} = 76.27p_1 = 919386kPa$$

Looking at figure 9.7 of the textbook, for $\theta = 27.5$, and $M_\infty = 3.5$ we see that $\beta = 44°$, we can use this to find the Mach number of the normal component of flow entering the shock wave.

$$M_{n,1} = M_1 \sin\beta = 2.43$$

Using appendix B of the text we can determine values after the shock

$$\begin{aligned} p_2 &= 6.722p_1 = 81027Pa \\ T_2 &= 2.069T_1 = 447.9K \\ \rho_2 &= 3.2489\rho_1 = 0.6303\frac{kg}{m^3} \\ M_{n,2} &= 0.52 \\ p_{02} &= 0.5302p_{01} = 487.458kPa \end{aligned}$$

M_2 is given by

$$M_2 = \frac{M_{n,2}}{\sin(\beta - \theta)} = 1.83$$

Again, Using appendix A, with $M_2 = 1.83$

$$\begin{aligned} T_{02} &= 1.67T_2 = 747.9K \\ \rho_{02} &= 3.603\rho_2 = 2.271\frac{kg}{m^3} \end{aligned}$$

The percentage of total pressure lost across the shock wave is

$$\frac{p_{01} - p_{02}}{p_{01}} = 47\%$$

From figure 9.7 of the text, the minimum freestream Mach number required to maintain an oblique shock attached at the nose is $M > 2.3$ which corresponds to a velocity of $v > 678.4\frac{m}{s}$