2004 - AER 307 F - Test 1
Friday, October 8
Time: 90 minutes
Format: Closed-book, no aids, non-programmable calculators

(At sea level, the density of the standard atmosphere is $1.226\frac{\mathrm{kg}}{\mathrm{m}^3}$ and the temperature is 288°K. For air, R = $287\frac{\mathrm{m}^2}{s^2\,{}^\circ\mathrm{K}}$. For a thermally-perfect gas, $p = \rho RT$ and for a perfect gas, $a^2 = \gamma RT$.)

Last Name	
First Name	
Student Number	

Question 1	
Question 2	
Question 3	
Total	

1. Given the Navier-Stokes equations in the compact form:

$$\frac{\partial}{\partial t}\begin{bmatrix} \rho \\ \rho\vec{v} \\ \rho E \end{bmatrix} + \vec{\nabla}\cdot\begin{bmatrix} \rho\vec{v} \\ \rho\vec{v}\otimes\vec{v} + p\overline{\overline{I}} - \overline{\overline{\tau}} \\ \rho H\vec{v} - k\vec{\nabla}T - \overline{\overline{\tau}}\cdot\vec{v} \end{bmatrix} = \begin{bmatrix} 0 \\ \rho\vec{f}_e \\ W_f + q_H \end{bmatrix} \tag{1}$$

identify the flux and source terms as fluxes or sources of mass, momentum, or energy, and give the physical process described. **[20 marks]**

$\rho\vec{v}$	
$\rho\vec{v}\otimes\vec{v}$	
$p\overline{\overline{I}}$	
$-\overline{\overline{\tau}}$	
$\rho H\vec{v}$	
$-k\vec{\nabla}T$	
$-\overline{\overline{\tau}}\cdot\vec{v}$	
$\rho\vec{f}_e$	
W_f	
q_H	

2 (a). Define the Reynolds number, $\mathcal{R}e$. What is it a ratio of in terms of forces? [**4 marks**]

2 (b). Define the flight Mach number, $\mathcal{M}$. For a perfect gas, why is easier to break the sound barrier at a higher altitude? [**4 marks**]

2 (c). Starting from the Navier-Stokes equations, list the two key approximations that give you the full-potential equations. What are two options for making the full-potential equations linear? [**6 marks**]

2 (d). Starting from the Navier-Stokes equations, what is the key approximation that gives you the Euler equations? While this approximation virtually eliminates any possibility of drag prediction for two-dimensional flow, what drag-generating phenomenon can still be predicted and what form of the equations is necessary to predict it? What is the boundary condition for velocity at a solid body when using the Euler equations? [**6 marks**]

3 (a). Sketch the following following boundary layer profiles (y vs. u): (i) attached flow, (ii) flow at the point of separation, and (iii) separated flow. Note that y is the normal coordinate to the body surface. [**9 marks**]

3 (b). List four conditions for boundary-layer flow. [**4 marks**]

3 (c). Consider the boundary-layer momentum equation for steady incompressible flow in two dimensions:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\mathrm{d}p_e}{\mathrm{d}x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{2}$$

Assume the viscosity, μ, to be constant. What does this equation reduce to at a solid surface? Based on this result and your sketches in (a), is a boundary layer more likely to separate in a favourable pressure gradient (pressure decreasing) or an adverse pressure gradient? [**7 marks**]

(extra space)