AER 307S: Test 2

At sea level, the density of the standard atmosphere is 1.226 Kg/m^3, and the temperature is 288 ^{o}K. Also, $R = 287 \text{m}^2/\text{sec}^2\ {}^{o}\text{K}$. Recall that, for an incompressible flow, $v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}$, and $v_\theta = -\frac{\partial \psi}{\partial r}$. Circulation is given by $\Gamma = -\int_C \vec{v}.\vec{ds}$. The Kutta-Joukowsky theorem gives $L' = \rho U_\infty \Gamma$. Also $e = mc^2$.

1. Sketch a boundary-layer velocity profile (y vs. u) for an attached flow, a flow at the point of separation, and a separated flow. Now consider the boundary-layer momentum equation for steady incompressible flow in two dimensions:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

If the viscosity is constant, what does this equation reduce to at a solid surface? Based on this result and your sketches above, is a boundary layer more likely to separate in a favourable pressure gradient (pressure decreasing) or an adverse pressure gradient (pressure increasing)? Hint: consider the sign of $\partial^2 u/\partial y^2$ at the surface for each of your sketches. (20 marks)

2. The absolute value of the velocity and the equation of the streamlines in an incompressible velocity field are given respectively by

$$|\vec{v}| = \sqrt{4x^2y^4 + 4x^4y^2}, \qquad xy = constant$$

Find the Cartesian velocity components. Hint: although the stream function is constant along a streamline, the above equation does not imply that $\psi = xy$, only that $\psi = f(xy)$. (20 marks)

3. An incompressible velocity field is described by the stream function

$$\psi = 100r\sin\theta\left(1 - \frac{25}{r^2}\right) + \frac{628}{2\pi}\ln\frac{r}{5}$$

On the circle defined by $r = 5$, $\psi = 0$. Therefore the flowfield for $r \geq 5$ describes an incompressible flow over a cylinder. Find (a) the locations of the stagnation points, (b) the circulation around the body, (c) the velocity at *infinity*, and (d) the force acting on the body. (20 marks)

4. Consider the following: 1) an airfoil in a windtunnel through which sea level air is drawn into a vacuum tank, and 2) an airfoil moving through sea level air at 30 m/sec. In both cases, the relative velocity between airfoil and air at a given point is 60 m/sec and the air is incompressible. Find the static pressure at this point for the two cases. Hint: they are not equal. (10 marks)