AER 336S: Problem Set 2

Please use fortran or C for this assignment. As always, submit a thoroughly commented source code. You may use the plotting package of your choice (matlab, if you wish) and put your name on the figures.

1. Code up the classical fourth-order Runge-Kutta method and apply it to the ODE:

$$\frac{du}{dt} = \lambda u + ae^{\mu t}$$

with $\lambda = -1 + 5i$, $\mu = i$, $a = 10$, and $u(0) = 1$. Calculate the solution at $t = 2$ and determine the error by comparing with the exact solution. Produce a log-log plot of the error as a function of Δt and confirm that the results are fourth-order.

2. Solve the following equation numerically on the domain $0 \leq x \leq 1$ with boundary conditions $u(0) = 0$, $u(1) = 1$ and initial condition $u(x) = x$:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 6x$$

Use second-order centered differences on a grid with 40 cells, i.e., 39 interior nodes. Use the fourth-order Runge-Kutta method developed above to integrate in time. Choose a suitable value of Δt. Plot the solution at regular time intervals as it approaches a steady state.

3. Compute the steady-state solution to the PDE in question 2 using 1) the point-Jacoby method, 2) the Gauss-Seidel method, and 3) the SOR method with the optimum value of ω. Use the same initial condition as in question 2. Plot the logarithm of the L_2-norm of the residual versus the iteration count. Plot the converged solution.