A NEWTON-KRYLOV ALGORITHM

FOR

AERODYNAMIC ANALYSIS AND DESIGN

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PRIORITIES IN CFD I

- CAD to mesh

PRIORITIES IN CFD II

- turbulence
  - transition prediction
  - transitional flows

ALAN TURING, 1946

Ace [the computer] " ... would be well adapted to deal with heat transfer problems, at any rate in solids or in fluids without turbulent motion."

NEWS REPORT, 1946

"Revolutionary developments in aerodynamics, which will enable jet-planes to fly at speeds vastly in excess of that of sound, are expected to follow the British invention of 'Ace', which has been commonly labelled the electronic 'brain'."

ALAN TURING, 1947

"One point concerning the form of organization struck me very strongly. The engineering work was in every case being done in the same building with the more mathematical work. I am convinced that this is the right approach. It is not possible for the two parts of the organization to keep in sufficiently close touch otherwise. They are too deeply interdependent."

ALAN TURING, 1947

"The masters [programmers] are liable to get replaced because as soon as any technique becomes at all stereotyped it becomes possible to devise a system of instruction tables which will enable the electronic computer to do it for itself. It may happen however that the masters will refuse to do this. They may be unwilling to let their jobs be stolen from them in this way. In that case they would surround the whole of their work with mystery and make excuses, couched in well chosen gibberish, whenever any dangerous suggestions were made."

INTRODUCTION & MOTIVATION

- aerodynamic optimization algorithm searches the design space for the designer
- the designer poses the optimization problem
  - objectives & constraints
  - design space
- issues
  - flow solution algorithm
  - optimization strategy
  - geometry parameterization

FLOW SOLVER: INEXACT-NEWTON ALGORITHM

- discretized Navier-Stokes equations with a one-equation turbulence model:

$$R(Q) = 0$$

- inexact-Newton strategy (outer iterations)

$$||R(Q_n) + A(Q_n)\Delta Q_n|| \leq \eta_n ||R(Q_n)||,$$

- GMRES (inner iterations)

  - matrix-free matrix-vector products:

$$A(Q_n)v = \frac{R(Q_n + \epsilon v) - R(Q_n)}{\epsilon},$$

PRECONDITIONING THE LINEAR SYSTEM

- right preconditioning:

$$AM^{-1}Mx = b$$

- block incomplete lower-upper factorization with fill: ILU(p)

- based on an approximate Jacobian matrix

$$\epsilon_2^{left} = \epsilon_2^{right} + \sigma\epsilon_4^{right}$$

- parameters: p and σ

THE START-UP PHASE

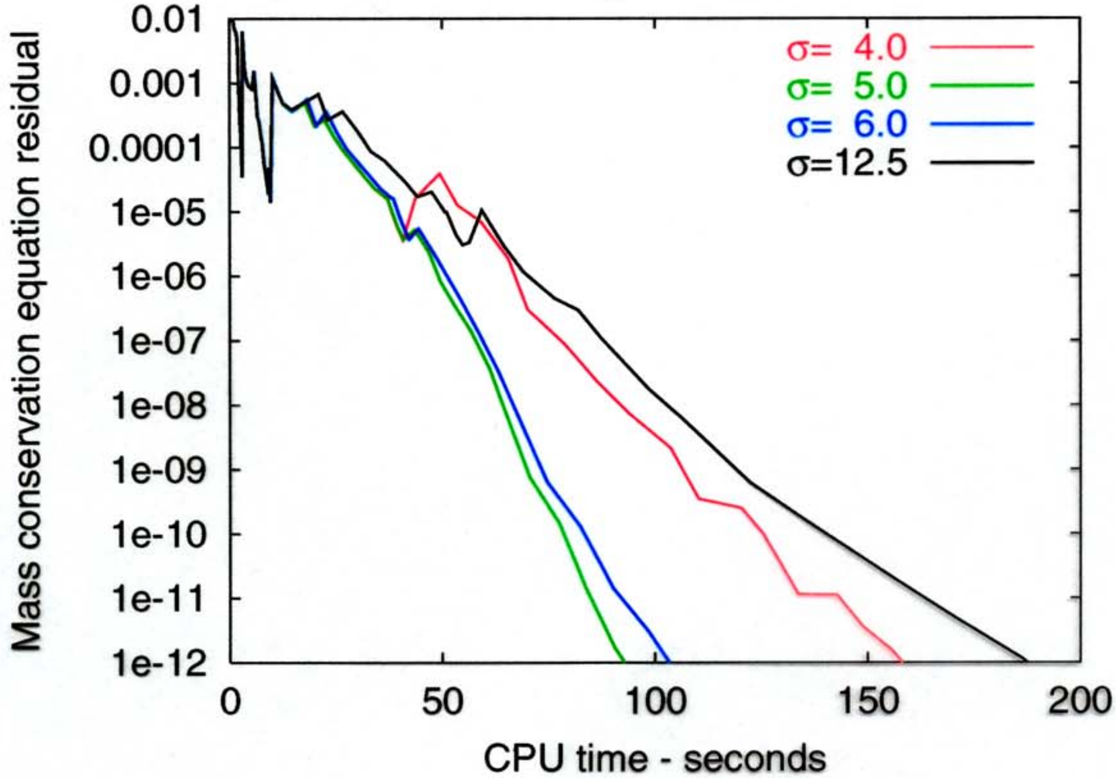
- difficulties:
  - turbulence model unstable for negative eddy viscosity
  - turbulence model slow to trip
- strategy:
  - finite time step initially:

$$\left(\frac{I}{\Delta t} - A_n\right) \Delta Q = R_n$$

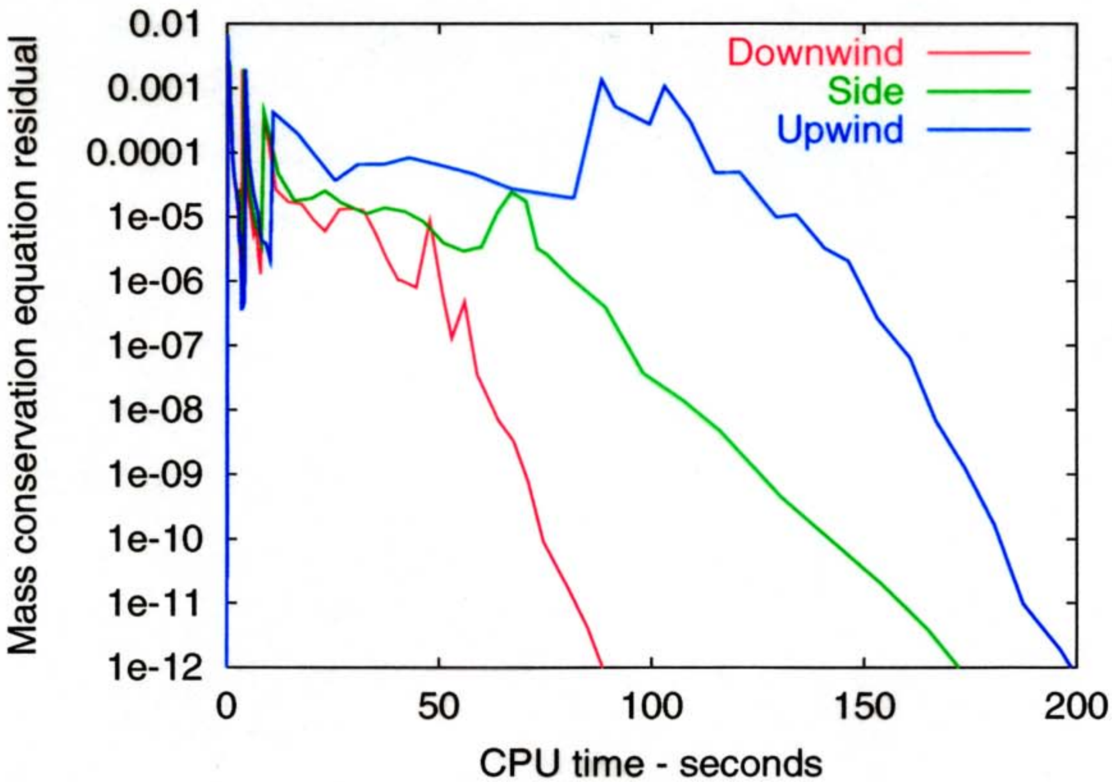
  - turbulence model Jacobian modified to produce an M matrix
  - linear residual reduced four orders of magnitude
  - grid sequencing

FLOW SOLVER SUMMARY

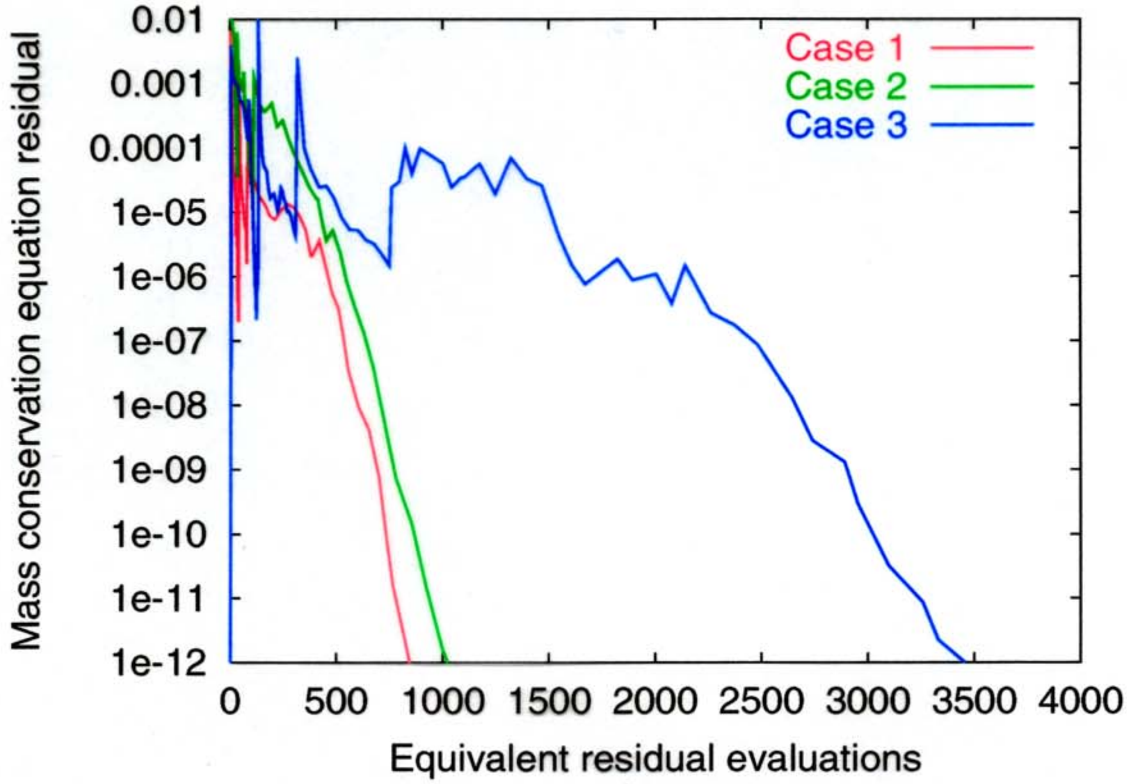
- compressible Navier-Stokes equations with Spalart-Allmaras turbulence model
- multi-block structured grids
- start-up and inexact-Newton phases
- linear solver: GMRES
  - matrix-free matrix-vector products
- right preconditioning: ILU(p)
  - approximate Jacobian matrix (σ)
- Reverse Cuthill-McKee reordering
  - choice of root node



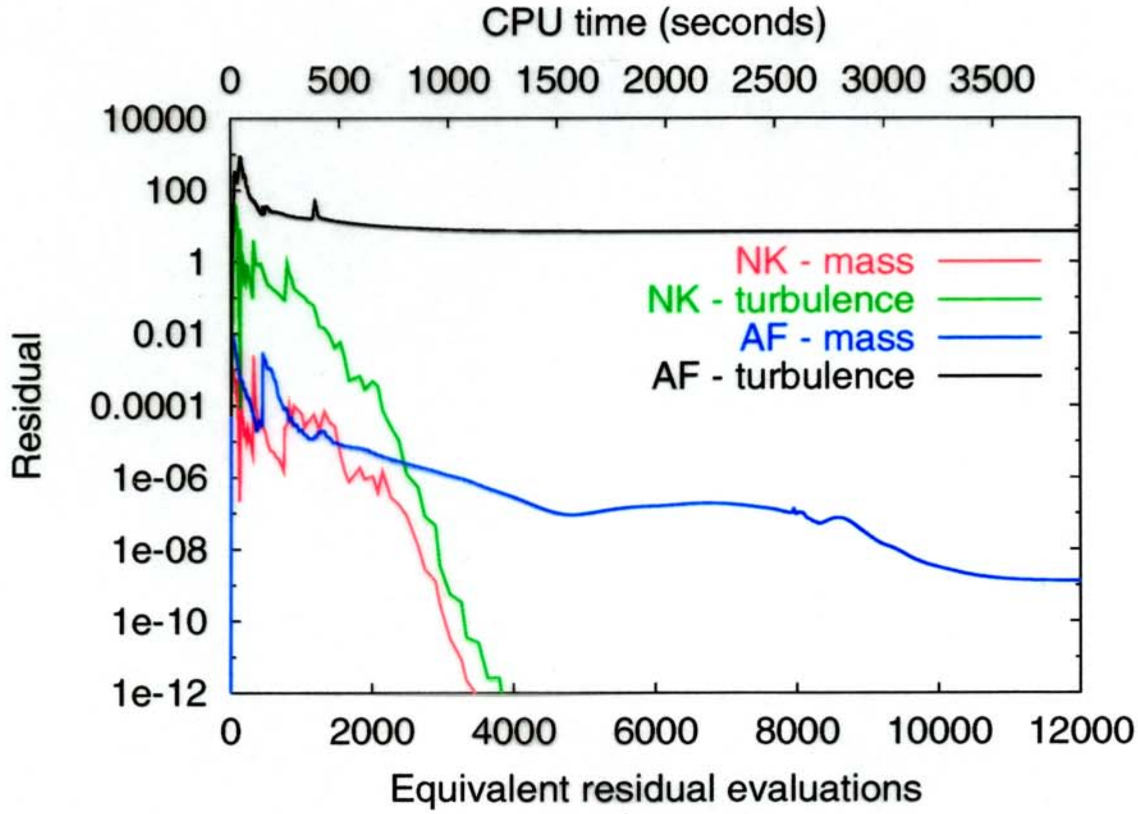
EFFECT OF σ PARAMETER



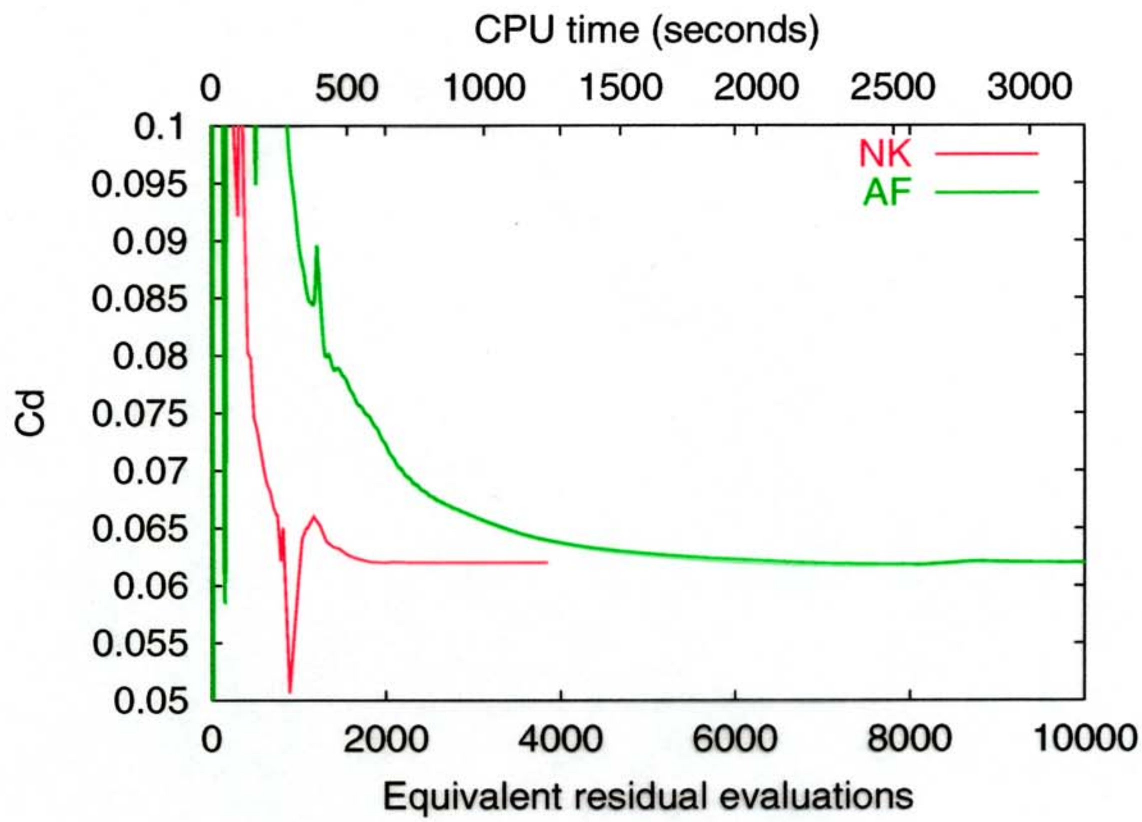
EFFECT OF RCM ROOT NODE LOCATION



CONVERGENCE HISTORIES
FOR THREE CASES

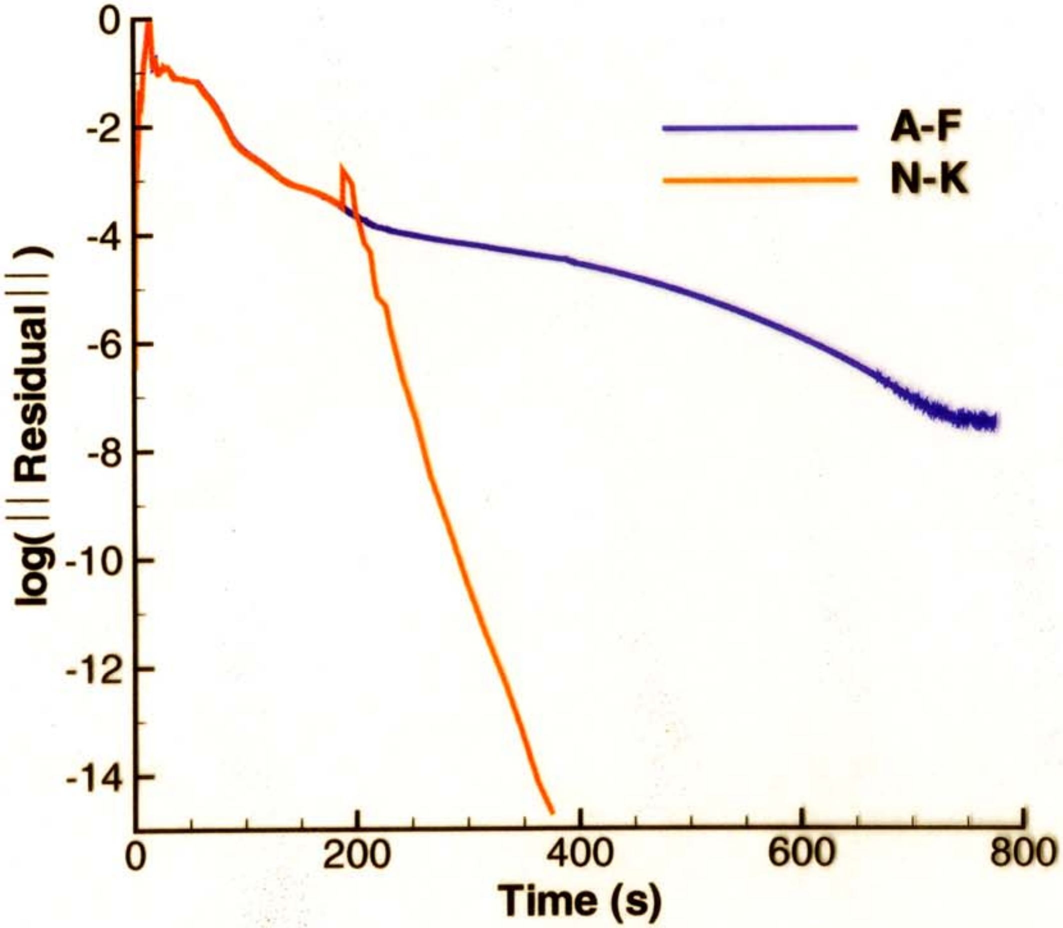


CONVERGENCE HISTORIES
FOR THREE-ELEMENT CONFIGURATION



DRAG CONVERGENCE HISTORIES
FOR THREE-ELEMENT CONFIGURATION

Flow Solver



NLR 7301, $M_\infty = 0.25$, $\alpha = 8°$, Re $= 2.51 \times 10^6$, Grid: 30,795 nodes

COMPONENTS OF A GRADIENT-BASED ALGORITHM FOR AERODYNAMIC DESIGN OPTIMIZATION

- geometry representation
- grid perturbation capability
- flow solver
- gradient calculation capability
- constraint formulation
- optimizer

Design Problem

design variables $X \longrightarrow$ shape of airfoil, α, gap, overlap ...

state variables $Q \longrightarrow$ density, velocity, pressure ...

objective or cost functional $\mathcal{J}\left[X, Q(X)\right]$

$\longrightarrow$ inverse design, drag, lift, moment, ...

$\longrightarrow$ multi-point and multi-objective design problems

constraints $\longrightarrow$ geometry constraints, such as thickness or area
$\longrightarrow$ flow equations and boundary conditions: $R\left[X, Q(X)\right] = 0$
$\longrightarrow$ flow constraints, such as C_{p} and C_{f}

Minimize $\mathcal{J}$, subject to satisfying $R = 0$
and any other side constraints.

Objective Functions

- Inverse design

$$\mathcal{J} = \frac{1}{2}\sum_{j=1}^{N_A}(C_{p_j} - C^*_{p_j})^2$$

- Lift-constrained drag minimization & lift enhancement

$$\mathcal{J} = \omega_{\mathrm{D}}\left(1 - \frac{C_{\mathrm{D}}}{C^*_{\mathrm{D}}}\right)^2 + \omega_{\mathrm{L}}\left(1 - \frac{C_{\mathrm{L}}}{C^*_{\mathrm{L}}}\right)^2$$

- Maximization of lift-to-drag ratio

$$\mathcal{J} = \frac{C_{\mathrm{D}}}{C_{\mathrm{L}}}$$

Constraints

- Airfoil thickness constraints are imposed using a quadratic penalty term:

$$\mathcal{J}_{\mathrm{C}} = \mathcal{J} + \mathcal{J}_{\mathrm{P}}$$

$$\mathcal{J}_{\mathrm{P}} = \begin{cases} \omega_{\mathrm{T}} \sum_{i=1}^{N_{\mathrm{T}}} \left(1 - \frac{t(x_i)}{t^*(x_i)}\right)^2 & \text{if } t(x_i) < t^*(x_i) \\ 0 & \text{otherwise} \end{cases}$$

- Similar formulation is used for gap and overlap constraints

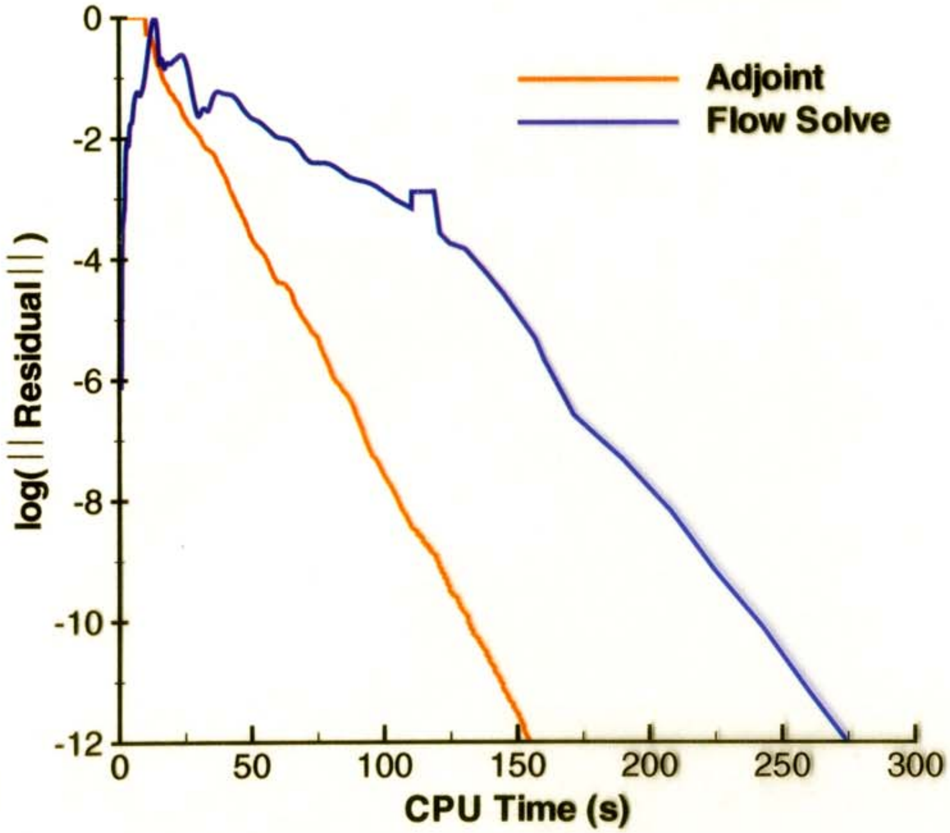
Quasi-Newton Method

- Unconstrained algorithm $\Rightarrow$ drives gradient to zero
- BFGS secant update and backtracking line-search procedure
- Input: objective function value and gradient vector
- Output: updated design variables

Gradient Computation:

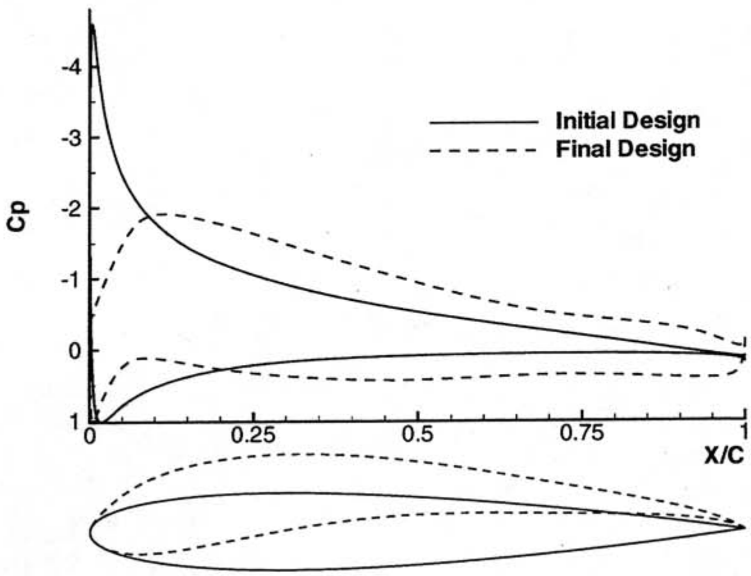
1. Finite-Difference Schemes
2. Flow-Sensitivity Method
3. Adjoint Method

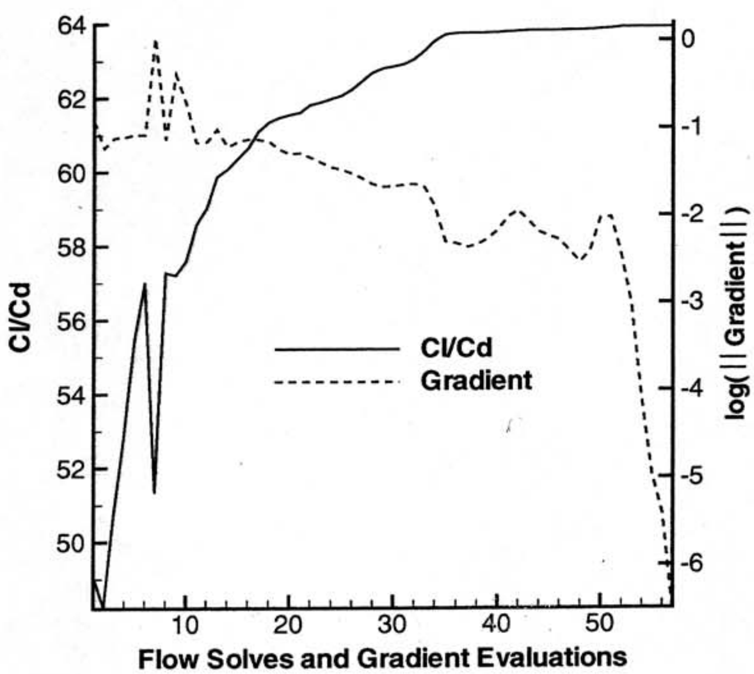
Gradient Evaluation



Lift Enhancement, NLR 7301 airfoil and flap

$M_\infty = 0.25$, $\alpha = 4°$, $Re = 2.51 \times 10^6$, Grid: 31,000 nodes

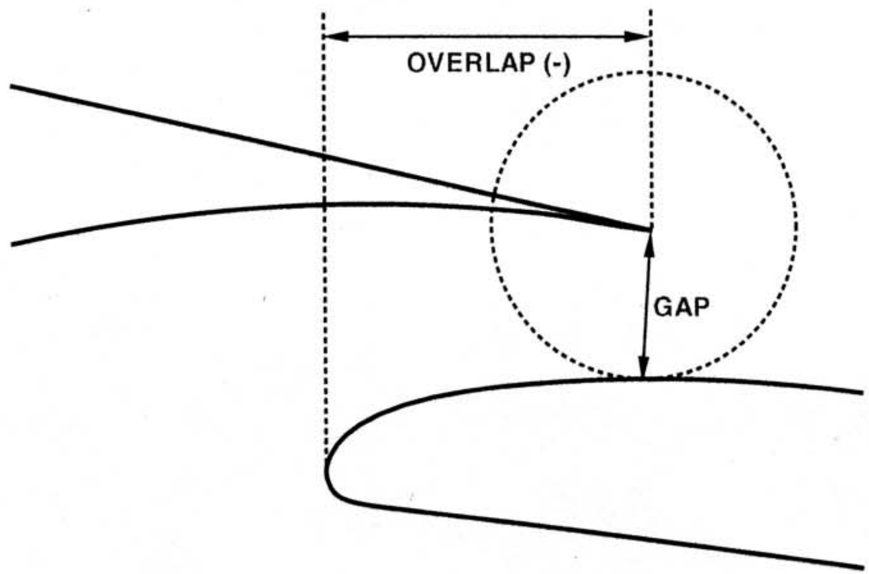




MAXIMIZATION OF LIFT-TO-DRAG RATIO

Lift-Enhancement Optimization of Gap and Overlap

- NLR 7301 Take-off Configuration
- Objective: Increase Lift and Maintain (or Lower) Drag



Gap Constraint: $\pm 0.5\%c$ Overlap Constraint: $\pm 1.0\%c$

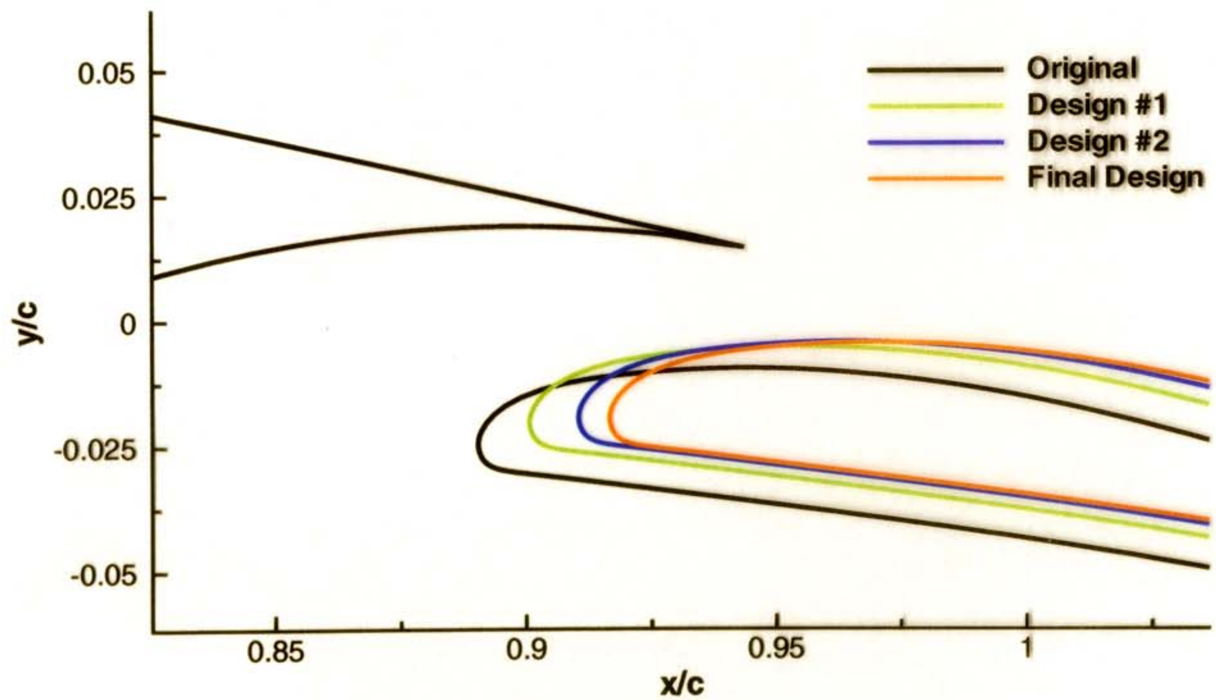
Gap & Overlap

Design	C_L	C_D	Gap $(\%c)$	Overlap $(\%c)$
Original	2.145	0.04720	2.40	-5.31
#1	2.165	0.04687	1.99	-4.28
#2	2.173	0.04677	1.95	-3.30
Final	2.175	0.04675	2.02	-2.68
Target	2.180	$\leq$ 0.04720		

Optimal Gap and Overlap Distances

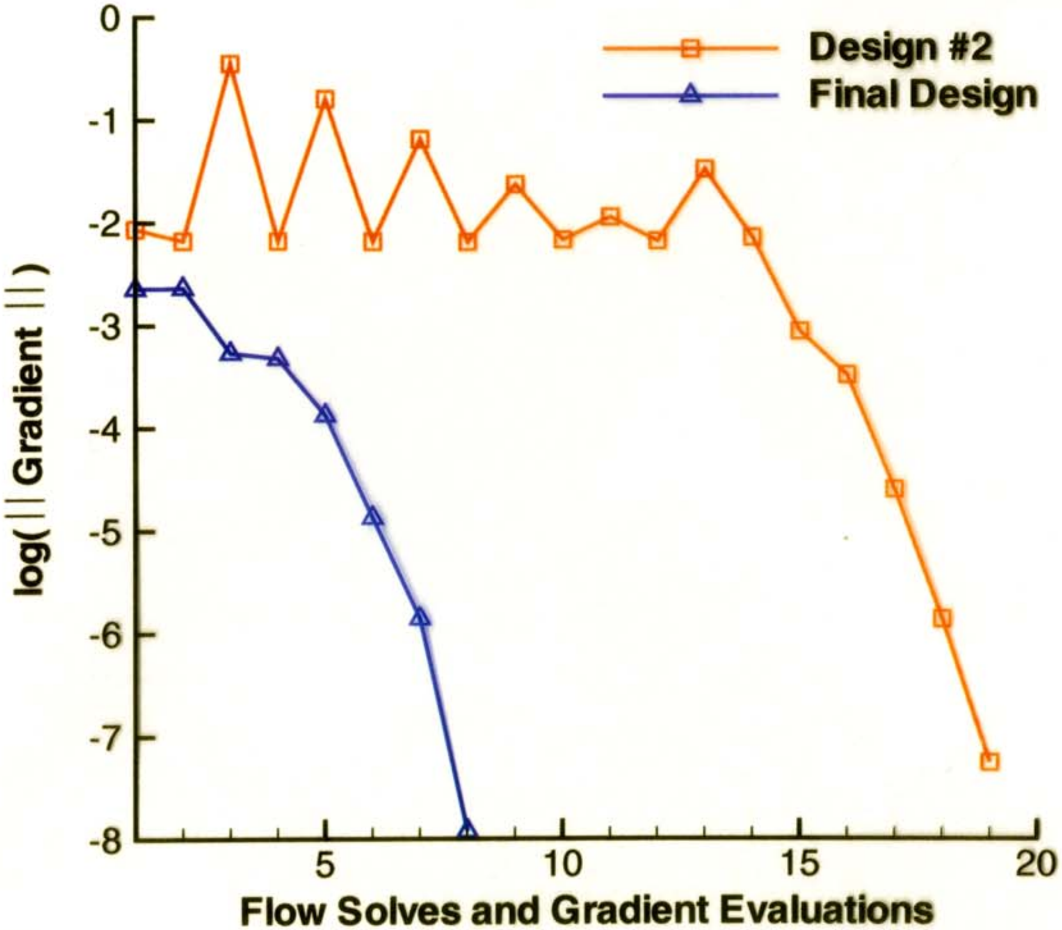
Gap & Overlap

- $M_\infty = 0.25$, $\alpha = 4.0°$, Re$= 2.51 \times 10^6$
- Grid: 31,000 nodes, off-wall spacing 2×10^{-6}



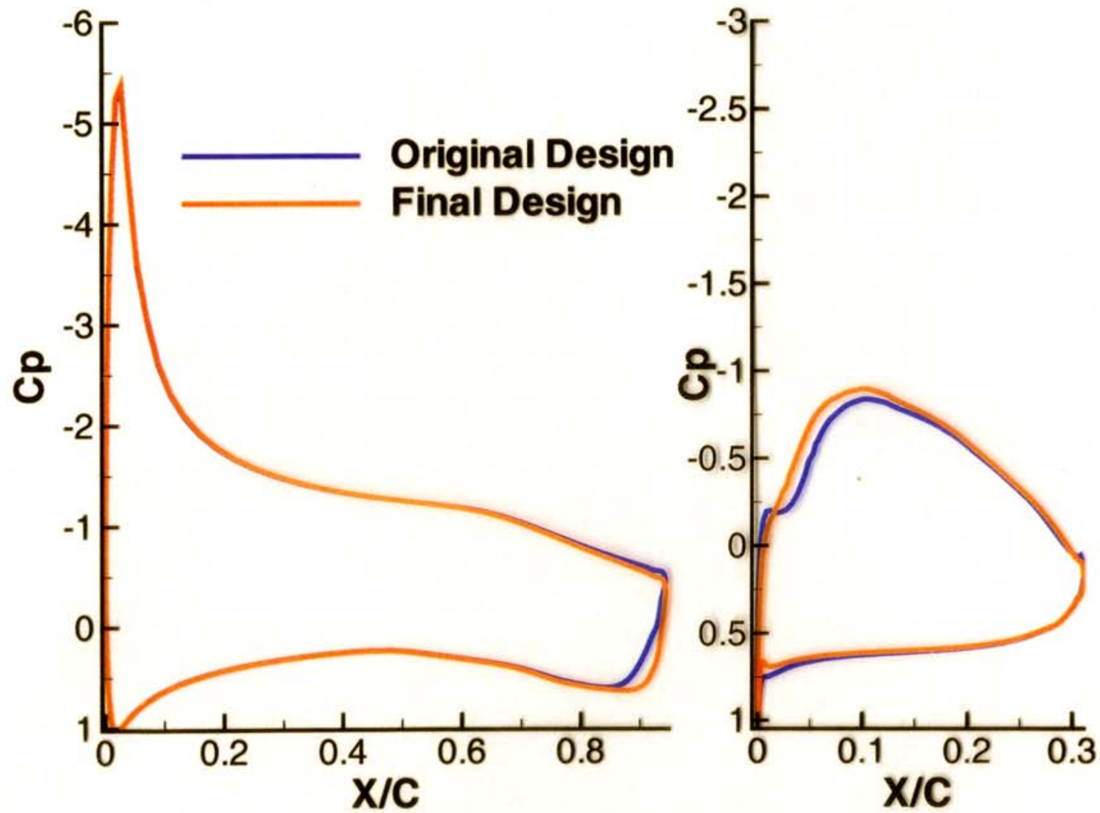
Flap Position History

Gap & Overlap



Optimization Convergence History

Gap & Overlap



Pressure distribution for the main element and flap

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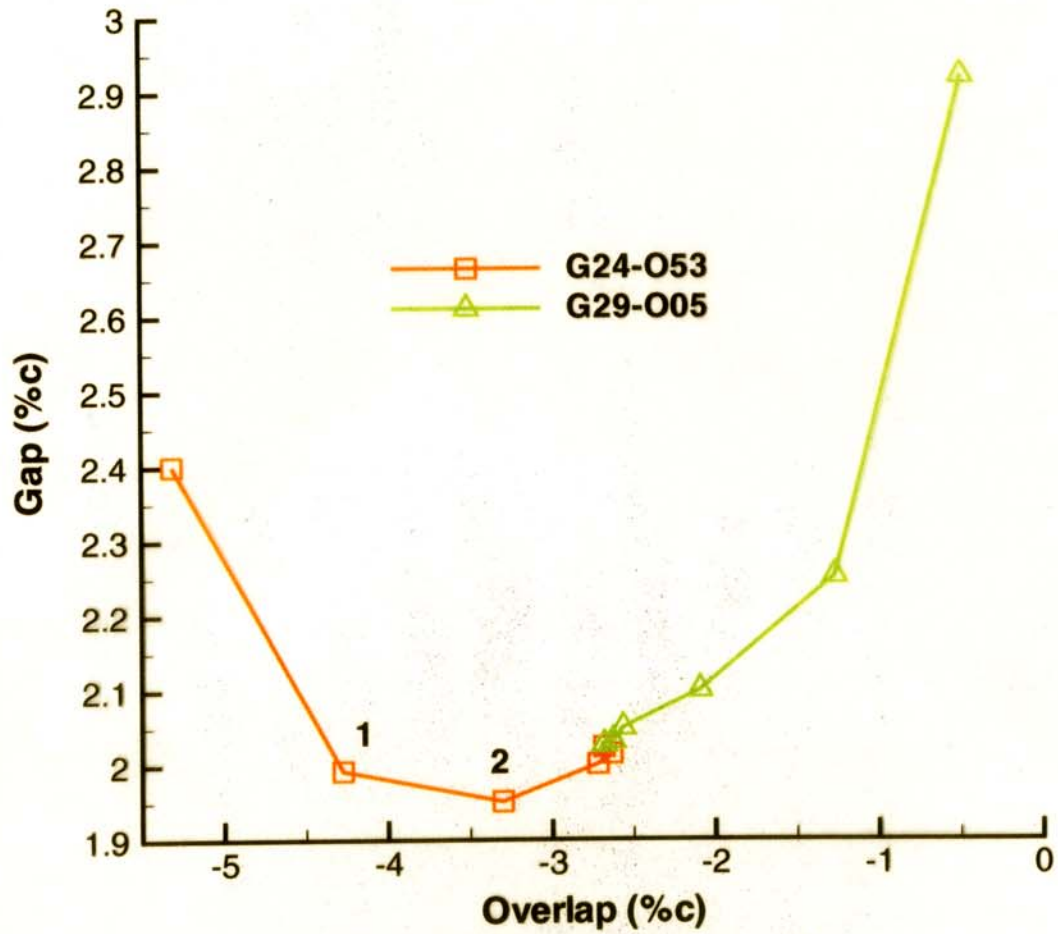
Gap & Overlap



Is the optimum unique?

PARETO OPTIMALITY

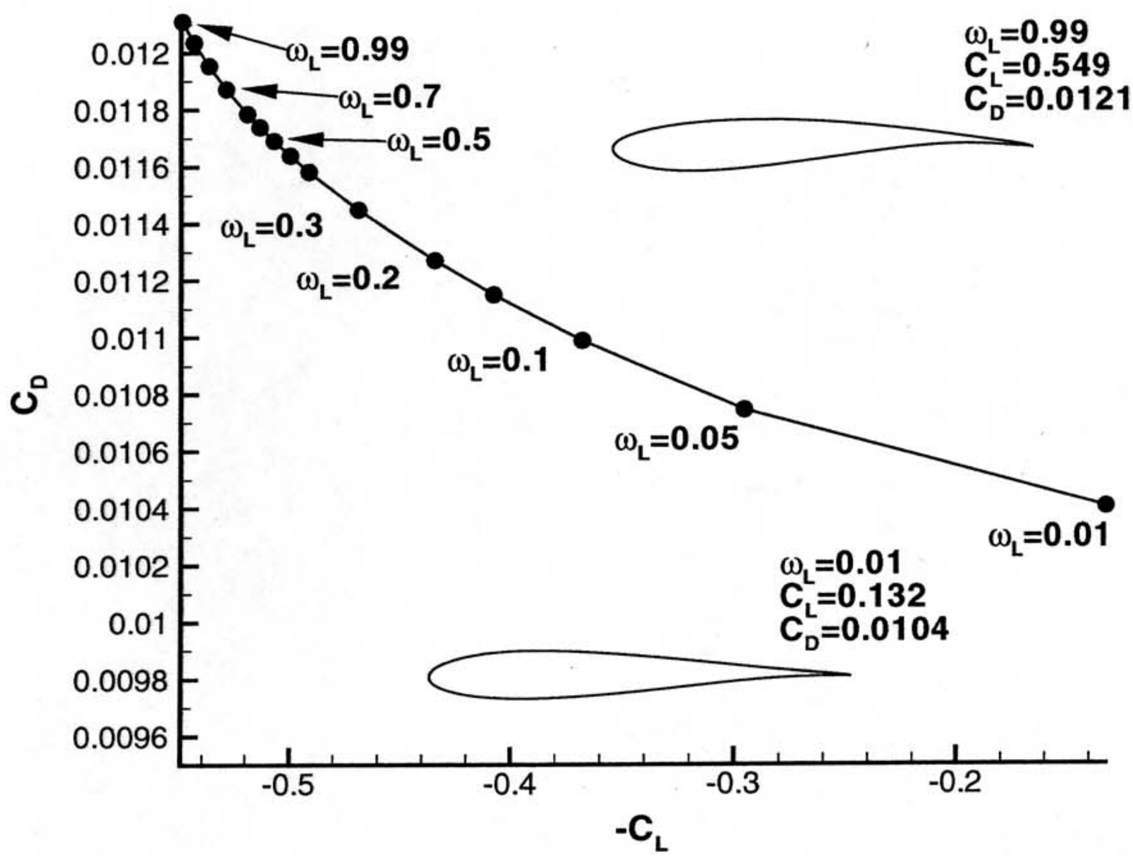
" ... an optimum allocation of resources is not attained in any given society when it is still possible to make at least one individual better off in his/her own estimation while keeping others as well off as before in their own estimation ... "

Vilfredo Pareto (1848-1923)

Multi-Objective Problems

- Consider design of high-lift and low-drag airfoils
- Define two objectives:
  1. $f_1 = \left(1 - \frac{C_\mathrm{D}}{C_\mathrm{D}^*}\right)^2$
  2. $f_2 = \left(1 - \frac{C_\mathrm{L}}{C_\mathrm{L}^*}\right)^2$
- Competition among objectives: $\Rightarrow$ there is no unique optimum
- We seek a set of non-inferior solutions: $\Rightarrow$ Pareto front
- *Weighted Sum Method*:

$$\mathcal{J} = (1 - \omega_\mathrm{L})\, f_1 + \omega_\mathrm{L} f_2$$



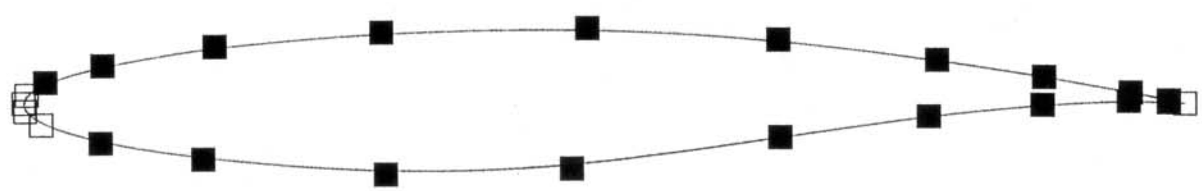
PARETO FRONT

FOR TWO COMPETING OBJECTIVES

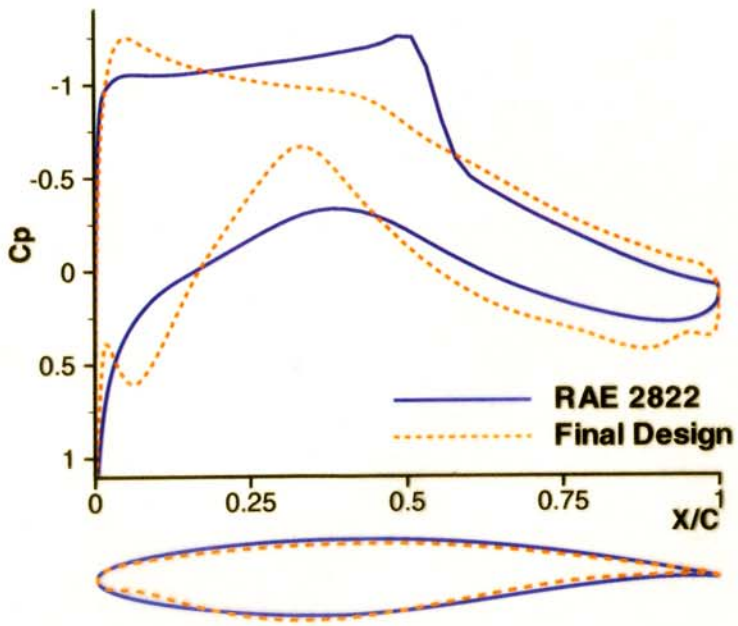
Lift-Constrained Drag Minimization for RAE 2822 Airfoil

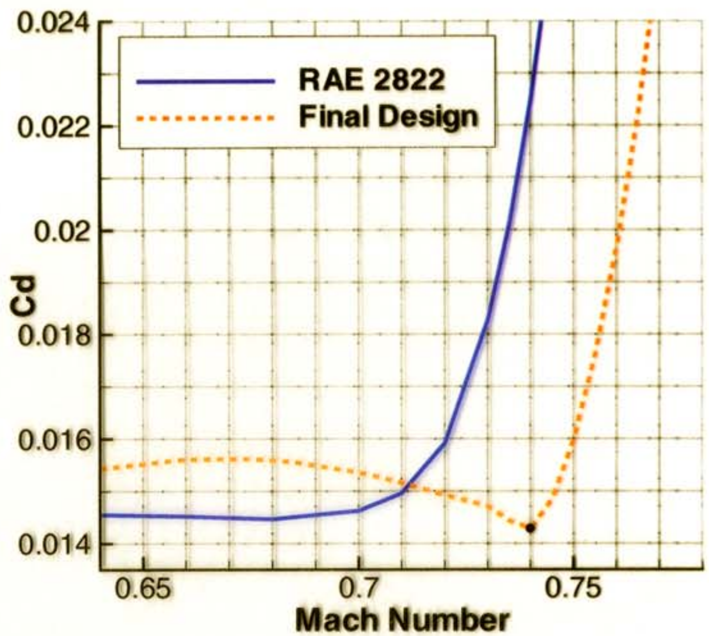
$$\mathcal{J} = \omega_{\mathrm{L}} \left(1 - \frac{C_{\mathrm{L}}}{C_{\mathrm{L}}^*}\right)^2 + \omega_{\mathrm{D}} \left(1 - \frac{C_{\mathrm{D}}}{C_{\mathrm{D}}^*}\right)^2 + \mathrm{T.C.}$$

- Design Point: $M = 0.74$, Re=2.7×10^6
- $\omega_{\mathrm{L}} = 1.0, \omega_{\mathrm{D}} = 0.1, \omega_{\mathrm{T}} = 1.0$, T. Cons. @ 0.35, 0.96, 0.99 %c
- Targets: $C_{\mathrm{L}}^* = 0.733$, $C_{\mathrm{D}}^* = 0.013$
- 19 Geometric Design Variables + α



One-Point Design





RAE 2822 $\alpha = 2.9^\circ$

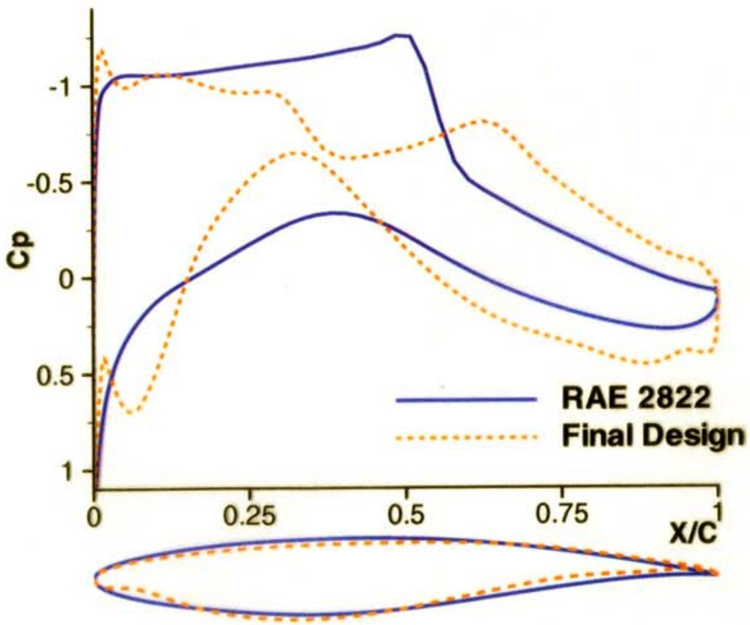
Final Design $\alpha = 1.9^\circ$

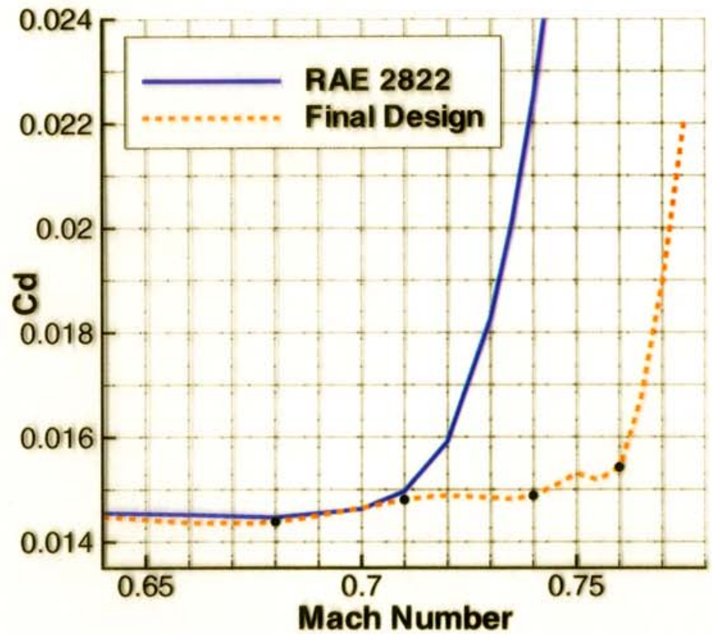
Drag Reduction

36.4%

Four-Point Design

$$\mathcal{J}_{\mathrm{MP}} = \frac{1}{7}\,\mathcal{J}\,(M = 0.68) + \frac{1}{7}\,\mathcal{J}\,(M = 0.71) + \frac{2}{7}\,\mathcal{J}\,(M = 0.74) + \frac{3}{7}\,\mathcal{J}\,(M = 0.76)$$





- For $M = 0.74$

Final Design $\alpha = 1.65°$ | Drag Reduction 33.8%

CONCLUSIONS & RECOMMENDATIONS

- The Newton-Krylov algorithm presented is a fast and reliable approach to aerodynamic analysis and design

- In order to guide further development, the algorithm should be applied to practical design problems characterized by complex multi-point objectives

- Optimum designs should be verified experimentally

- Effect of uncertainty needs to be quantified