

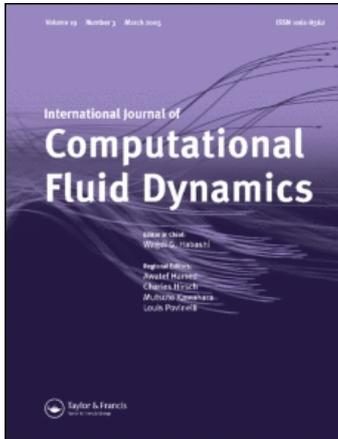
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A perspective on turbulence models for aerodynamic flows

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Turbulence modelling options are discussed in the context of steady aerodynamic flows. After a brief overview of popular turbulence models, four criteria are presented that should be satisfied in order to conclusively evaluate a turbulence model with respect to its ability to predict a specific flow. Many past studies do not meet these criteria. This is followed by some sample results for several turbulence models, including one-equation, two-equation and algebraic Reynolds stress models. The three main conclusions are as follows. First, more combined experimental–numerical studies are needed that meet the four criteria for assessment of turbulence models. Second, of the models studied, the Spalart-Allmaras model provides the most accurate results for the high-lift flows examined. Finally, the most significant factor limiting our present ability to predict many aerodynamic flows accurately is our inability to reliably predict laminar-turbulent transition. Until this issue is addressed, the benefits of an improved turbulence model will be limited.

Keywords: turbulence models; aerodynamics; computational fluid dynamics; Reynolds-Averaged Navier-Stokes equations; turbulent flows; multi-element airfoils; high lift; algebraic Reynolds stress models

1. Introduction

Rising fuel costs and the pressing need to reduce greenhouse gas emissions per passenger-kilometre from aircraft have led to renewed interest in drag reduction. Development of drag reduction technology is greatly aided by aerodynamic shape optimisation based on computational fluid dynamics (CFD), which is dependent on the capability to predict aerodynamic flows accurately. Aerodynamic flows are characterised by high Reynolds numbers, typically ten to one hundred million, Mach numbers ranging from low subsonic at take-off and landing to supersonic, and a combination of laminar and turbulent flow. Such flows are typically attached or mildly separated and steady, with large-scale separation or unsteadiness present under limited circumstances, such as in coves, behind deployed spoilers, under post-stall conditions, after the onset of buffet and other off-design conditions. Prediction of aerodynamic flows requires the ability to compute phenomena such as boundary layers, wakes, confluent boundary layers, shock-boundary-layer interactions, laminar-turbulent transition, transitional flows, separation points, separated flows and reattachment points.

In the context of aerodynamic shape optimisation, given the status of present-day computers and algorithms, there is currently no practical alternative to solving the Reynolds-Averaged Navier-Stokes

(RANS) equations. The RANS equations include the effects of turbulence through Reynolds stresses, which are apparent stresses that arise as a result of time-averaging the Navier-Stokes equations over a time interval much longer than the characteristic time scales of the turbulence. This brings us to the subject of this article, turbulence models, which are models for the Reynolds stresses needed for closure of the RANS equations.

Historically, the evolution of turbulence models has progressed in the following manner. First a model of a given complexity is introduced. The earliest models represent the Reynolds stresses as an algebraic function of the velocity field and its gradient. These models are calibrated and shown to lead to accurate predictions of a given suite of flows. Inevitably, the model has limitations, and it is extended to flows for which inaccurate comparisons with experimental data are obtained. Typically, a few particularly difficult flow problems are identified. These serve as the catalyst for the next generation of turbulence models, which are typically of increased complexity. The superiority of the new models is often demonstrated by their accuracy for these specific problems.

This evolutionary process is flawed in two important respects. First, it is often possible to tune a turbulence model to achieve a specific known result. Therefore, it is insufficient to demonstrate that a model

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is more accurate for a specific problem without showing that accuracy equivalent to or better than that of the previous models is maintained for a broad suite of flows. For example, a turbulence model that produces a maximum lift coefficient that is lower and in better agreement with experiment than other models for a specific airfoil may produce a maximum lift coefficient that is too low for other airfoils for which the other models are accurate. Second, the flow problem for which inaccurate comparisons with experiment are obtained may be difficult for reasons unrelated to the turbulence model. It could be that the experimental data are flawed. For example, a purportedly two-dimensional data set could actually have three-dimensional features, thus causing the discrepancies assumed to be the result of an inadequate turbulence model. There are several possible reasons for disagreement between theory and experiment other than the turbulence model, as further discussed below.

The objective of the present article is to provide a perspective on turbulence modelling for aerodynamic flows based on the authors' combined thirty 30 years of experience solving such flows. As such, we do not provide a comprehensive overview of available turbulence models. Our goal is to provide some thoughts relevant to the choice of a turbulence model and some future research directions, rather than recommending a specific model. The reader is referred to Wilcox (2006) for a thorough treatment of turbulence models, to Rumsey and Ying (2002) for a comprehensive discussion of various turbulence models applied to computations of high-lift flows, and to Godin (2004) for more details of the results presented here. Finally, we restrict our attention to steady flows, as there currently exists too little data relating to unsteady flows to draw conclusions.

2. Turbulence models

In CFD, turbulence can be handled in several different ways, depending on the nature of the problem, the computing resources available, the required turnaround time and the accuracy needed. The present hierarchy includes the RANS approach, detached eddy simulation or other hybrids involving the RANS approach with large eddy simulation, large eddy simulation and direct numerical simulation. Relative to the other approaches, the RANS equations can be solved on coarser meshes and permit the simplification of steady flow. Consequently, solving the RANS equations is currently the only viable option for most practical aerodynamic computations.

Within the RANS approach, turbulence models must meet several criteria. First, they must be sufficiently accurate for a specific class of flows of

interest. Next, they must be affordable in the context of the day. Finally, their numerical properties must be appropriate. For example, a model should not require the use of a mesh that is substantially finer than that needed for the mean flow, it should be able to be implemented on arbitrary meshes, including unstructured meshes, which means it should be local in nature, and it should not have a substantially adverse effect on the convergence of the solver.

Turbulence models range in cost and complexity from simple algebraic models, through scalar evolution models, to expensive Reynolds-stress models. Algebraic models have well-known limitations (Spalart and Allmaras 1994), while Reynolds-stress models have not demonstrated the definitive improvement in accuracy needed to justify their increased expense and complexity. Therefore, most efforts have been towards scale evolution models in which partial differential equations are solved for certain scalar quantities, and the Reynolds stresses are then calculated based on the mean flow field and the scalar quantities. Most such models rely on the eddy-viscosity or Boussinesq approximation, which uses an analogy to the relation between stress and rate of strain in laminar flows to define an effective or eddy viscosity. This approximation defines the Reynolds stresses in terms of two scalar parameters, the eddy viscosity and the turbulence kinetic energy. The kinetic energy term is often dropped, as it does not affect the shear stress that is dominant in thin shear flows, and it is somewhat approximate (Spalart and Allmaras 1994).

Popular eddy-viscosity models include the algebraic model of Baldwin and Lomax (1978), the one-equation model of Spalart and Allmaras (1994), and the two equation models of Launder and Spalding (1974), Wilcox (1988), and Menter (1994). The $k-\varepsilon$ model of Launder and Spalding has been the dominant turbulence model in CFD in general but has not received much acceptance in the aerodynamics community because of its questionable accuracy in predicting aerodynamic flows, especially high-lift flows, and its numerical properties. The original $k-\omega$ model of Wilcox has an improved near-wall treatment compared to the $k-\varepsilon$ model but has also performed relatively poorly in aerodynamic flows. Menter's model combines the two in an attempt to address their limitations and, in addition, makes use of Bradshaw's shear stress transport (SST) assumption. Evidence suggests that Menter's model is more accurate than the original $k-\omega$ model (Godin *et al.* 1997). A revised $k-\omega$ model (Wilcox 2006) has yet to be thoroughly tested. For computations of aerodynamic flows, the Baldwin-Lomax model was the dominant turbulence model throughout the 1980s. Despite its limited capability to accurately handle shock-

boundary-layer interactions, confluent boundary layers and wakes, its simplicity and accuracy led to its popularity. In an attempt to address these deficiencies, while retaining the computational advantages, Baldwin and Barth (1990) introduced a one-equation model derived from the k - ε model. This was closely followed by the Spalart-Allmaras model, which has been shown to perform well in several studies (Fejtek 1997, Godin *et al.* 1997, Nelson *et al.* 1998). This model has its shortcomings, as will be discussed later, but has arguably become the turbulence model of choice for aerodynamic flows. Godin *et al.* (1997) concluded that the Spalart-Allmaras model is superior for general computations of aerodynamic flows, whereas the Menter SST model is preferred if separated flows are of primary interest.

Although eddy-viscosity models have been used with success for wall boundary layers and free shear flows, Bradshaw (1973) has observed that the Boussinesq hypothesis fails for boundary layers over curved surfaces and for separated flows. To accurately predict curvature effects, turbulence models must be capable of realistic representation of all components of the Reynolds stress tensor, not just the shear stresses. In order to deal with the anisotropy associated with curved flows while avoiding the cost of a Reynolds-stress model, non-linear eddy-viscosity and algebraic Reynolds-stress models (ARSMs) have been introduced. ARSMs use non-linear algebraic expressions to compute the Reynolds stresses, based on a hypothesis originally proposed by Rodi (1976). Good results have been obtained for high-lift flows (Davidson 1991, Stolcis 1992), but ARSMs have not become popular due to their cost and poor numerical properties. There are two major types of ARSM, implicit (IARSM) and explicit (EARSM). Although their origins are quite different, they differ primarily in the solution algorithm for the resulting algebraic relationships and the pressure-strain correlation. The more traditional IARSM formulation has gradually given way to the EARSM formulation, which has increased robustness (Speziale 1997). Although all ARSMs are based on the non-linear algebraic Reynolds-stress closure, they vary in terms of the models used for the pressure-strain correlation and the Reynolds-stress dissipation rate tensor as well as the near-wall treatment. Moreover, different models can be used to determine k and ε , such as the k - ε and k - ω models.

3. Turbulence model assessment

In this section, we present four criteria which must be satisfied in order to conclusively evaluate a turbulence model with respect to its ability to predict a specific flow. These criteria are rarely satisfied, and this has

often led to erroneous conclusions with respect to the accuracy of turbulence models and has hampered the development of a good understanding of the capabilities and deficiencies of turbulence models in general.

First, numerical errors must be reduced to a level where they are significantly smaller than the turbulence modelling errors under study. This ensures both that the numerical and turbulence model errors do not compound one another and that the discrepancies between numerical predictions and experimental data are caused by the turbulence model rather than by numerical error. Historically there have been many cases where the turbulence model was a convenient scapegoat when in fact the culprit was inadequate mesh resolution. Ensuring that the numerical error is much smaller than the turbulence modelling error also prevents false positives where the two errors cancel to produce good agreement for a flow where the turbulence model is inaccurate. Although this criterion is easy to state, it can be quite difficult to achieve, especially in three-dimensional computations (Mavriplis *et al.* 2008). Although they play an important role, grid convergence studies do not necessarily provide proof that a numerical solution is grid independent. In high-Reynolds-number turbulent flows, features can vary in size by several orders of magnitude. Therefore, if a flow feature is completely unresolved by a given mesh, then refining by say a factor of two may be insufficient to resolve the feature. Hence the more refined mesh may give the same result as the coarser mesh, leading to the erroneous conclusion that the numerical error is very small. Another important issue to keep in mind is that some numerical errors are not related to grid resolution; these cannot be reduced by refining the grid. Examples include the thin-layer approximation and the effect of the outer boundary. Zingg (1992) and Fejtek (1997) have both noted the significant effect of the location of the outer boundary and the far-field boundary condition itself on the prediction of drag.

Second, the experimental errors must be significantly smaller than the turbulence modelling errors. There is no point in attempting to evaluate turbulence models by comparing with experimental data that contains errors larger than the differences in the predictions of the turbulence models. In comparisons with computational results, there often seems to be an underlying assumption that the experiment has no error. However, experimental errors can be significant, and this has also led to misleading conclusions related to the accuracy of turbulence models. Studies of high-lift multi-element airfoil flows near maximum lift provide a particularly important example. The paper by Rumsey *et al.* (2003) should be required reading for anyone interested in this issue. Motivated by

discrepancies between two-dimensional computations and experiments, it demonstrates the difficulties involved in maintaining two-dimensional flow under high-lift conditions and in recognising that the flow is not two-dimensional. Rumsey *et al.* (2003) also show that the flow can be very sensitive to details of the experiment, such as side-wall venting, mounting brackets, as well as tunnel disturbances and asymmetries. Three-dimensional computations still show differences from the experimental data, and there remain enough uncertainties that one cannot draw conclusions about the role of the turbulence model in contributing to these discrepancies. In flows where such difficulties are not present, one can expect accuracy of $\pm 1\%$ in mean velocities, roughly $\pm 5\%$ in fluctuating components, and closer to $\pm 10\%$ in $\overline{u'v'}$.

Third, the computation must represent precisely the same geometry and flow conditions as the experiment. Although closely related to the second criterion, we distinguish them to emphasise the difference between experimental error and the need to precisely measure and define all of the quantities needed by the computational model. In many experiments, data which could be important, such as upstream flow conditions, tunnel turbulence levels and background noise, are either not measured or not reported. For example, the angle of incidence of an airfoil can change once the airfoil is loaded; if this is not measured, then it leads to a potential uncertainty in the computation. Once again, the study by Rumsey *et al.* (2003) provides several examples. They point out that the differences between their three-dimensional computations and the experimental data could be caused by numerical error (our first criterion), physical differences between the computational model and the experiment (our third criterion), such as the absence of top and bottom tunnel walls as well as mounting brackets in the computation, or modelling errors, including the turbulence model but also the boundary condition used to represent the side-wall venting. They also raise the possibility that the flow could be unsteady in the corner flow separation regions, which is not represented in the computations, since they assume steady flow. Their study shows that it can be very difficult to draw definitive conclusions about the ability of a turbulence model to predict a specific flow even with a very carefully combined experimental–numerical study.

Fourth, the location of laminar-turbulent transition must be known or predicted accurately. Many turbulent flowfields, such as high-lift flows, depend critically on the nature and location of transition and the transitional flow region. Yet the phenomenon of transition is rarely given much attention in RANS solvers, and our ability to predict transition in a manner that is compatible with a RANS solver (as opposed to a

boundary-layer solver) lags our turbulence modeling capabilities. In many cases, inaccurate prediction of transition is a far greater impediment to accurate flowfield prediction than shortcomings in turbulence modelling. This criterion can be generalised to state that all physical model errors other than the turbulence model error must be reduced to sufficiently low levels in order that the turbulence model error can be properly identified. However, prediction of transition is of particular importance and is thus emphasised.

4. Results

In this section, we show some sample results from the study of Godin (2004) in order to illustrate the predictive capabilities of several representative turbulence models. Note that we do not claim that these comparisons meet the four criteria described in Section 3. They represent our best efforts with regard to numerical accuracy and available experimental data. Transition points are set based on experimental evidence when available. Godin considered four eddy-viscosity models, including two one-equation models, Baldwin-Barth and Spalart-Allmaras, and two two-equation models, $k-\omega$ and Menter SST. In addition, he studied both explicit and implicit ARSMs driven by the $k-\omega$ model with a variety of pressure-strain correlation models and near-wall treatments. It is important to note that the original $k-\omega$ model was used for these studies, not the revised one. On the basis of preliminary studies on a single-element airfoil at a high angle of incidence with separated flow over the aft 20% of the upper surface and a multi-element configuration at a modest angle of incidence, four turbulence models were chosen for further study, one from each category. The Spalart-Allmaras model (SA) slightly outperformed the Baldwin-Barth model; the SST model was substantially more accurate than the $k-\omega$ model (which predicted the separation point much too far aft) for the separated flow case. The explicit ARSM (EARSM) selected is based in Girimaji's solution algorithm (Girimaji 1996) with the pressure-strain correlation model of Speziale *et al.* (1991). No additional near-wall treatment is needed, as this pressure-strain correlation model includes the anisotropic dissipation term. The implicit ARSM (IARSM) chosen uses the pressure-strain correlation model of Gibson and Launder (1978) and near-wall bridging based on Bradshaw's approximation (Menter 1994).

We now show results obtained using the four selected turbulence models for two multi-element configurations. The first is the NLR 7301 airfoil and flap configuration examined experimentally by Van den Berg (1979). The particular case shown here

in Figure 1 has an angle of incidence of 13.1° and a gap of 2.6% chord. A small movement of the flap relative to its nominal position was noted during the experiment; the measured position was used for the computations. The Reynolds number is 2.51 million, and the Mach number is 0.185. The structured multi-block grid used has a total of 182,295 nodes with an off-wall spacing of 10^{-7} chords. Grid refinement studies indicate that this mesh provides sufficient resolution for our purposes here. Transition was fixed on both elements based on experimental observations. Figure 2 shows the experimental and computed surface pressure coefficient distributions. All four turbulence models produce excellent agreement with the experimental data. Figure 3 displays velocity and Reynolds shear stress profiles at four stations. The first station is on the upper surface of the main airfoil near the trailing edge. The predictions of the four models vary significantly, with the SA model producing the best agreement with the experimental data in both the velocity profile and the Reynolds shear stress. The remaining three stations are located on the flap; thus they show both the flap boundary layer and the wake of the main element. For these stations, the SA model is also the most accurate in predicting the velocity profiles and the Reynolds shear stress. It is the only model of the four that correctly captures the evolution of the lower portion of the main-element wake. The

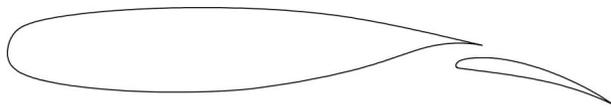


Figure 1. NLR 7301 airfoil and flap.

two ARSMs display a strange behaviour in the Reynolds shear stress profiles in the wake of the main airfoil at the furthest downstream station, despite providing more reasonable results a short distance upstream. Further study is needed to examine the cause.

Next we examine Case A-2 from Moir (1994), which was the focus of the benchmarking exercise summarised by Fejtek (1997). This is a three-element geometry with a leading-edge slat and a single-slotted flap, as depicted in Figure 4. The case we consider has an angle of incidence of 20.18° , a slat angle of 25° and a flap angle of 20° . The Reynolds number is 3.52 million, and the Mach number is 0.197. The grid has 134,051 nodes and was carefully generated to minimise discretisation error. The normal spacing at the surface is such that y^+ values are well below unity everywhere. The mesh stretching ratio is another important parameter. A ratio close to unity tends to reduce numerical errors and ensures that boundary layers are sufficiently well resolved. The present mesh stretching ratio is 1.18. Laminar-turbulent transition is assumed to occur just after the point of minimum pressure on each element. As shown in Figure 5a all of the models predict the surface pressure coefficients very well. In Figure 5b total pressure profiles are displayed at four stations, two on the main element and two on the flap. There is considerable variation among the models, and none track the experimental data particularly well. The discrepancies at the outer edge of the slat wake are almost certainly due to inadequate grid resolution (see Hellsten 2005). One could argue that the SA model predictions are closest to the experimental data based primarily on the flap stations, but overall all of the models provide comparable accuracy for this case.

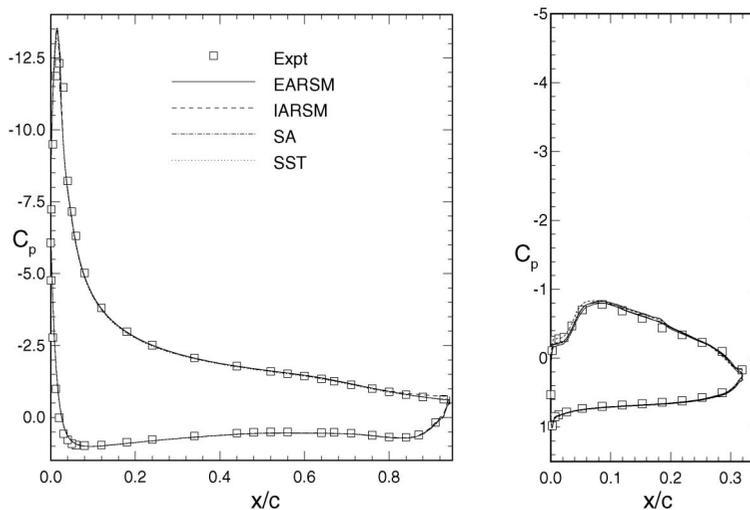


Figure 2. Surface pressure distribution.

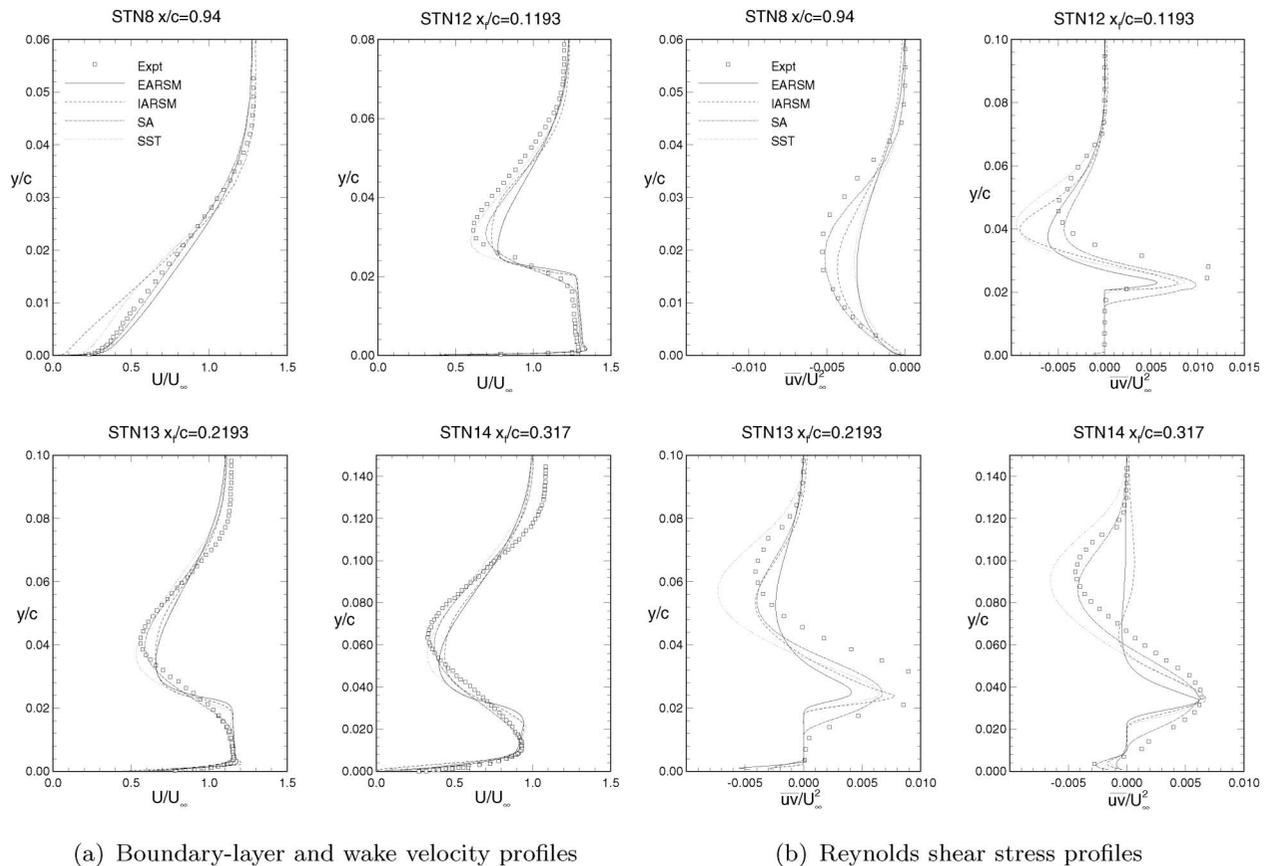


Figure 3. Comparison of results for the NLR7301 airfoil with 20-deg flap and $\alpha = 13.1^\circ$.

5. Discussion

The sample results presented here are typical for such flows. For many cases, the eddy-viscosity models (SA and SST) are found to outperform the ARSMs. While this is perhaps surprising, given the expected advantage of the ARSMs in curved flows, it strongly suggests that the eddy-viscosity approximation is not among the leading sources of error. The advantage of the eddy-viscosity models, especially the Spalart-Allmaras model, is most apparent when confluence is a dominant flow feature. The ability to predict the maximum lift coefficient of a high-lift configuration depends heavily on the location of laminar-turbulent transition. Near maximum lift, transition generally occurs via laminar separation and reattachment. Most RANS solvers are not capable of predicting the details of such a laminar separation bubble. Moreover, two-dimensional experimental data tends to become suspect at maximum lift.

In examining results such as these, it is important to keep in mind that, of the four turbulence models compared, the Spalart-Allmaras model is the simplest to implement and the least expensive. Considerable effort was expended developing a robust iterative



Figure 4. Three-element configuration.

algorithm to stabilise the ARSMs, which required a converged mean-flow and turbulence field, given by the prior solution of a two-equation model, as initial conditions (Godin 2004). Given the low computational expense of the Spalart-Allmaras model, one would require conclusive evidence that another model is significantly more accurate. On the contrary, we find the Spalart-Allmaras model to be the most accurate of those studied in most contexts. The most notable exception is in the prediction of flows with large regions of separated flow, where the Spalart-Allmaras model tends to predict separation bubbles that are too thin. The Menter model and both ARSMs provided better predictions of a separated flow studied by Godin (2004). However, there are again questions about the two-dimensionality of the experimental data, so further study is needed.

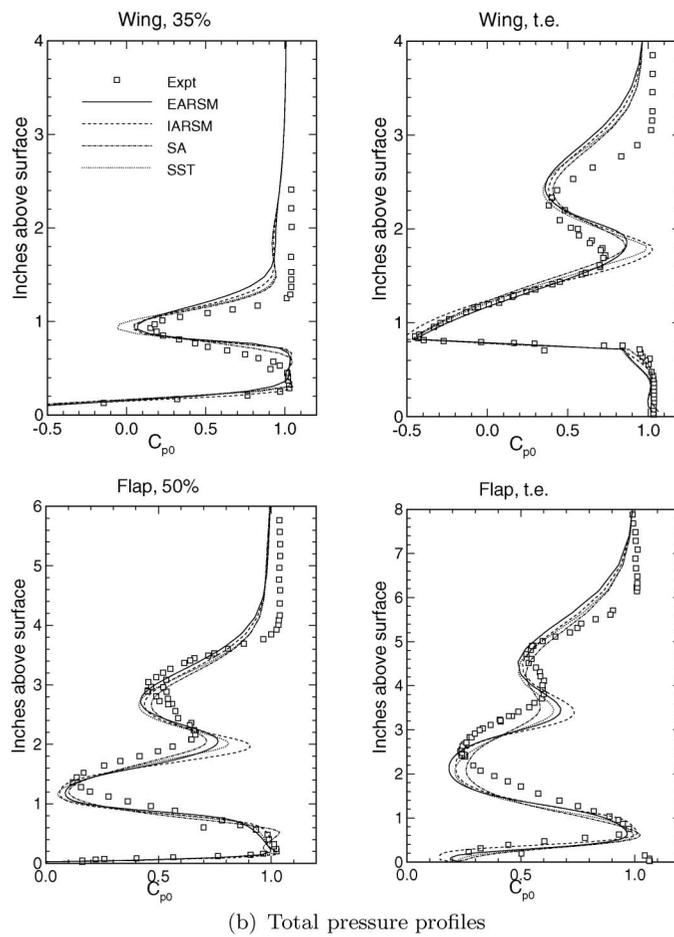
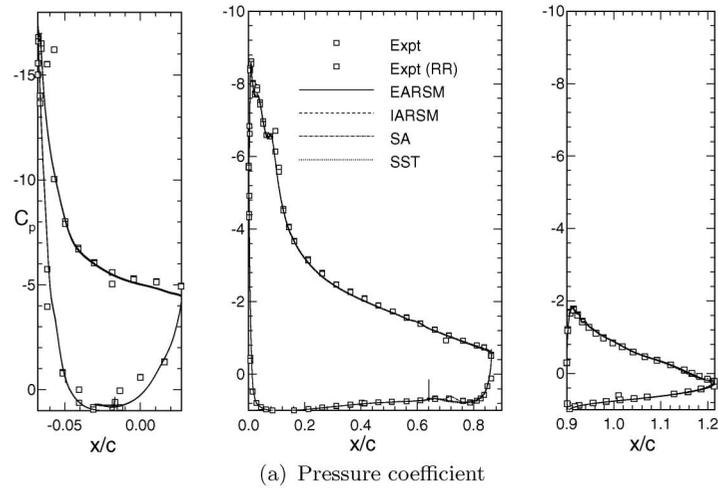


Figure 5. Results for A2 case at 20 degrees angle of attack.

The role of streamline curvature in these flows is difficult to assess. The wakes show some asymmetries that could be related to curvature, but the ARSMs do not appear to capture these asymmetries more accurately than the eddy-viscosity models. The Spalart-Allmaras model has been modified to account for

streamline curvature effects (Spalart and Shur 1997), but these modifications have not been incorporated here.

In the application of CFD to design and optimisation, it is critical that the accuracy of the computations be sufficient to predict the subtle effects resulting from

small variations in parameters. For the NLR 7301 two-element configuration discussed above, experiments were performed with two different settings of the gap between the main element and the flap. The measured difference in the lift coefficient between the two flap settings is 0.050. Using the Spalart-Allmaras model, a difference of 0.027 is computed, while the Menter model leads to a difference of 0.005, the IARSM a difference of 0.014 and the EARSM a difference of 0.006. These results reflect the superiority of the Spalart-Allmaras model in handling the merging of boundary layers and wakes. However, even the Spalart-Allmaras result differs significantly from the measured difference in lift, which indicates that perhaps none of these models is sufficiently accurate to optimise the location of a flap.

These turbulence models have received little validation in unsteady flows, so their applicability for such flows is not well understood. It appears that the Spalart-Allmaras model introduces too much eddy viscosity in large-scale flow structures that produce unsteadiness, thus suppressing the unsteadiness. This has led some authors to switch off the production terms in such regions, e.g. Khorrami *et al.* (2002). Although this can produce a degree of agreement with experimental data, it must be considered an *ad hoc* correction at this stage.

On a side note, in order to maintain positivity of the eddy viscosity, the convective terms in the Spalart-Allmaras model are usually discretized based on first-order upwinding. This raises the question whether this becomes the leading source of error in a solver that is nominally second-order or higher. The work of De Rango and Zingg (2001) sheds some light on this issue. They were able to substantially reduce numerical error by raising the order of all terms in the spatial discretisation except the convective terms in the Spalart-Allmaras model from second-order to at least third-order. This indicates that the first-order approximation for the turbulence model convective terms is not adding significantly to the overall error.

6. Conclusions

This study attempts to convey the following basic ideas. First, in order to perform meaningful evaluation of turbulence models by comparison with experimental data, one should ensure that discrepancies arising from the following four sources be at least an order of magnitude less than the anticipated turbulence model errors:

- (1) Numerical error, including errors that depend on grid refinement and those that do not,
- (2) Experimental error,

- (3) Differences in the geometry and flow conditions between the experiment and the computation,
- (4) Errors from physical models other than the turbulence model, especially the prediction of laminar-turbulent transition.

Many past studies do not meet these criteria. Unless they are met, there can be too much uncertainty to draw definitive conclusions from the comparison.

Next, based on accuracy and numerical properties, it has been our experience that the Spalart-Allmaras model is the best available option for many aerodynamic flows, providing accurate predictions of attached and mildly separated flows. Its superiority is particularly evident when confluence of boundary layers and wakes is important. For flows with large-scale separation, the Menter SST model may be the better choice, but there is still some uncertainty due to the difficulties in obtaining reliable experimental data for such flows.

Finally, given the reasonable accuracy provided by the Spalart-Allmaras and Menter SST models when the location of laminar-turbulent transition is known as well as the sensitivity of these flow fields to the transition location, the most significant factor limiting our present ability to predict many aerodynamic flows is not the turbulence model but our ability to reliably predict transition. We strongly encourage further efforts to develop accurate transition prediction techniques that are compatible with RANS solvers (e.g. Langtry and Menter 2005, Cliquet *et al.* 2008).

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